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# Water table shape and flow nets at the upper end of subsurface drains

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WATER TABLE SHAPE AND FLOW NETS AT THE  
UPPER END OF SUBSURFACE DRAINS

by

Ross Weston Irwin

A Thesis Submitted to the  
Graduate Faculty in Partial Fulfillment of  
The Requirements for the Degree of  
MASTER OF SCIENCE

Major Subject: Agricultural Engineering

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Signatures have been redacted for privacy

Iowa State College

1954

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## INTRODUCTION

Of the water that falls upon the earth as rain a greater or lesser part percolates into the soil to become subsurface water. Some of the subsurface water is taken up by the plants to be transpired by them through their leaves. A portion is evaporated directly; and some, the hygroscopic water, resists evaporation and is held in the soil. The free water moves downward and laterally under the influence of gravity until it reaches an outlet; in the case of this study it is a tile drain.

The control of the water level in the soil at a point where it is satisfactory for plant growth is one of the most important factors in successful crop production. The ground water level or water table continually fluctuates with the seasons and is near the surface in spring when the young plants require a sufficient depth of unsaturated soil in order to get a healthy start. To attain this aim, under-drainage has come to be recognized as one of the most valuable practices. It has been proved by scientific study and practical observation that the benefits to be derived from well drained farm land are numerous (31).

The ground water level is lowered between tile lines in a curved line, the highest point being midway between the drains. Field crop roots can neither live nor obtain

food from a saturated soil. They require a well aerated drained soil for sustenance. The height to which a water table will rise, together with the rate at which it is lowered after a rain, is one measure of the efficiency of operation of an underdrainage system. Schlick (63) states that as long as the tile has sufficient capacity the ground water level is controlled by the spacing and depth of the tile drains. With drainage the shape of the water table is changed (11). The effect of lowering the level of the water table is more pronounced immediately over the tile drains than at some distance from them. The spacing between the tile lines should be such that the water table midway between the lines is not more than one or one and a half feet above the flow line in the tile drain (60, p. 376). This factor is not readily predetermined, but depends on the various soil physical conditions, such as permeability as discussed by Frevert (21). Many people have put forth various formulae to express this drain spacing factor mathematically in terms of some soil property that is readily measured. This work has been excellently reviewed and presented by van Schilfgaarde (77).

Muskat (49, p. 8) states that

. . . while it is true that the indetermining of certain conditions involved in problems of flow through porous media encountered in practice will prevent the attainment of exact

quantitative results from any mathematical analysis, it is nevertheless certainly of value to analyse the problems as if they involved ideal systems. For it is only in this manner that the fundamental properties of the system can be determined and their behaviour under modified conditions ascertained.

Drainage research is faced with two different but related problems: the practical one of determining the optimum design for a drainage system on any given piece of land, and the more purely scientific one of determining how the system functions, and what effect it has on the soil and on the crops. Russell (62) remarks on the element of uncertainty which prevents optimum design from being determined beyond a certain degree of accuracy. He lists these elements as weather, change of farming methods, nature of the soil, and economics.

The purpose of this study is to determine the water table surface at the upper end of subsurface drains by means of flow nets and water table observations. The problem includes the development of a proper procedure, design and construction of equipment, installation of the experiment, and analysing the data. The aim in under-draining is to get the most drainage per unit cost and secure the highest possible efficiency for every foot of tile used. The importance of the end of a tile drain lateral has been overlooked in the past. With an increased

knowledge of this aspect of a tile system some economies in the use of tile might be effected.

The upper end of lateral drains influence the layout of experimental plots. The tendency at the present time is to extend the drain well beyond the last plot. It is possible that with some new knowledge on this subject a more efficient use may be made of the existing experimental land. The basic requirement would be a uniform water table throughout the length of the plots at the midspacing.

The upper end of each lateral drain is blocked off by some method. This is common practice and apparently the custom is to use whatever material is readily available. Up to the present time there has been very little study given to this phase of drainage practice.



## REVIEW OF LITERATURE

Interest has been centered on drainage from before the time of Columella in Rome to the present time and a great volume of work has been done in the many phases of drainage research. Much of this work has been duly recorded in the many fine text books, bulletins, and journal papers on the particular problem investigated. In the study of the effect on the water table near the upper end of a tile drain the paucity of references in the literature pertaining to this subject has been a cause of much concern.

There are a great many methods available to the investigator for determining flow nets or patterns in order to secure a solution to a flow problem. These may be classified in the following manner:

- a. Mathematical solution by hodographs as reviewed by Muskat (49, p. 300).
- b. Sand or glass beads in a tank and using piezometers in a wide tank (10) or dye streams in a narrow tank (25).

- c. Trial sketching of equipotential and streamlines as discussed by Forchheimer (18).
- d. Experimental mathematics by the iteration method where relaxation is a special case of this type of mathematics and was used by Shaw and Southwell (69) and Luthin (42). This method is tedious. The mathematical solutions are generally used for checking or academic interest and for a few simple cases where equations for the boundaries can be derived.
- e. Electrical analogies which will be discussed in more detail.
- f. Piezometer tubes in the field are of low precision as it is not easy to get controlled conditions.

### Electrical Analogy

Recent studies of drainage problems have shown the use of the electrical analogy to be very important in overcoming certain of the problems. Wyckoff (82, p. 395) states that

. . . problems involving the flow of homogeneous fluids through porous media under the action of gravity in general require very difficult analytical methods for solution. Furthermore the

problems may become entirely intractable mathematically when the geometry of the flow system takes on only a reasonable complexity such as is involved in many systems of practical engineering interest.

He states that scale models have been used in the past but due to the difficulty of making them sufficiently accurate he suggests that

. . . a method based on the electrical analogy of flow of fluids through porous media which is simple but nevertheless capable of providing reasonably accurate and detailed solutions regardless of the complexity of the problem

could be used.

Hubbard and Ling (29) discuss the accuracy of the electric analogue and state that experiments on several analogues of axially symmetrical flow systems in which a comparison could be made with analytical methods, it appeared safe to use the electrical analogy to represent a design problem in which the absence of an axis of symmetry made mathematical analysis impractical or virtually impossible. In organizing and investigating the shape of the water table at the upper end of a tile drain it was desirable to analyse and use much of the literature dealing with the application of the electrical analogy to this particular problem.

In the past there has been much accomplished by use of the electrical analogy. Reltov (58) states that the

method of electrical analogy was proposed by Prof. N. N. Pavlovsky in 1920 for investigating problems of ground water flow under dams. Using electric piezometers buried in the base of his powdered graphite model, Reltov in 1933 made use of a three-dimensional case and established that models with different scales give similar results. Reinius (56) and Casagrande (4) employed it on dam seepage. Vreedenburg (78) for further work on dams used a paraffin model with copper electrodes in a diluted salt solution which had its depth proportional to the permeability coefficient of the soil. Muskat (49, p. 455) referred to a three-dimensional analogue in a water and oil problem through the same media. Bradfield (3) developed an electrical tank but used it to solve a three-dimensional torsion problem. Wyckoff (82) working on seepage under dams developed a colloidal graphite gel on heavy paper to vary the permeability and determine the top seepage line by a cut and try method. Childs (7) used the electric analogy very extensively to determine drain flux and flow patterns under several different conditions. A different model was used for each geometrical configuration, spacing and depth. He determined only the equipotential lines and traced the streamlines by trial sketching.

In 1948 Frevert (20) using tap water as an electrolyte

successfully inverted the analogue to determine a geometric factor for permeability studies. This is apparently the first time both equipotential lines and streamlines were determined by this method. Schwab (66) used this method to determine the flow into circular perforations and Dutz (12) used the same method for determining the flow into cracks between adjacent tiles. Many other workers determined needed factors with the electric analogy. Van Bavel and Kirkham (75), Luthin and Kirkham (43), and Maasland (47) used it for phases of soil permeability studies.

Stenström (71) made a very precise model for investigating low velocity air distribution around a three-dimensional aircraft model. Relf (57) also used it for aircraft work. McClelland (44) gives a very excellent review of the graphite method for two-dimensional problems. A new approach was used by Luthin (41) in electrical resistance networks to solve soil and water problems. This method was first advanced by Liebman (39). Lane (37) working on a study of Boulder Dam used tinfoil in place of carbon for the model.

The limitations of the method of electrical analogy for the solution of problems of fluid flow is that a stationary water table is required where the free surface is an equipotential line, a condition rare in actual

practice although Childs (8) (9) developed a method for a moving water table. The ponded condition is only temporary but is the maximum condition to be expected so the method is quite applicable. The assumptions made are that Darcy's law is valid, the soil is homogeneous, and the tile is running full without back pressure. In the case under consideration this is not achieved as the upper end of a tile drain will be nearly empty. In the analogue uniform conductivity is easily achieved but does not correspond to the permeability in the field where there are varying soil types and disturbed soil in the tile trench. The conducting material must possess sufficient degree of electrical conductivity and uniformity and should not change with time.

### Field Study

At increasing and low velocities porous flow is purely viscous with a gradual transition toward turbulence as the velocity exceeds a critical value for the particular media. This is assumed to be a Reynolds number of less than one which gives it application to any soil finer than coarse sand, assuming spherical particles (49, p. 67). The permeabilities of the porous media encountered in underground flow is so low that any departure from viscous flow involves pressure gradients which are out of the range of those ordinarily encountered. Through isotropic



soils the velocity of flow is expressed by Darcy's equation:  $v = K i$  where  $v$  is the velocity of flow,  $K$  is the hydraulic conductivity, and  $i$  is the hydraulic gradient.

Muskat (49, p. 140) presents a table illustrating the similarity and relationships between the flow of water, electricity, heat and magnetism. Electricity is more convenient to handle experimentally. There is an analogy between Ohm's law for electrical flow and Darcy's law for fluid flow in the steady state. The quantity of fluid flow  $Q$  is equivalent to the current  $I$  in electrical flow. The permeability  $K$  represents the specific conductivity and the hydraulic head corresponds to the voltage. Advantage is taken of this analogy to solve hydrodynamic problems by exploring the distribution of potential in an electrolyte of uniform depth contained in a tank.

#### Flow into drain

Tile drains are laid in the soil in a continuous line and upon such a grade that any water which finds its way into the tile through the cracks (88) between individual tile will be carried away under the influence of

gravity to some lower point where it is discharged. A careful search of the literature has revealed but a single reference regarding the amount of flow into the upper end of a tile drain.

Pickels (52, p. 180) states:

. . . the ground water enters the drain at the joints. The water which enters the first joint at the upper end of a lateral is very small - probably about 2 per cent of the capacity of the drain - and hence the hydraulic grade line may be assumed to start at the elevation of the invert of the drain. At the succeeding joints approximately equal inflows will enter the drain . . . ; but these inflows will be less than that at the first joint, since it serves a much wider strip above the end of the lateral.

The above value of 2 per cent was probably arrived at by using a ration of the surface area of the tile to a 1/8-inch crack spacing. This would be a volume basis only, without respect to time. Kirkham established (34) that a 1/8-inch crack gave a flow resistance of 56 per cent when compared to a totally open drain for a common situation.

#### Area drained

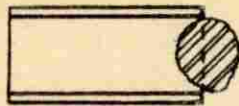
There is very little information on the area that the upper end of a tile drain will drain. Schroeder (66, p. 211) states that the upper end of a tile drain will "drain" a distance equal to one-quarter the spacing



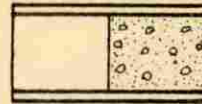
because the laterals drain an approximate semicircular area off the end. Etcheverry (14, p. 89) states that at the upper end of a lateral the only flow is that from a small area adjacent to it. Although they do not discuss it, Etcheverry (14, p. 100) and Fauser (15, Fig. 12) presenting illustrations of particular tile layouts, indicate that a header drain (a drain essentially at right angles to parallel lateral drains and a short distance above the upper end of them) should be placed a distance of one-half the normal spacing above the end of the lateral drains. This would indicate they do not feel the ends of the laterals would drain a very large area. Schroeder (65, Fig. 147) for similar layouts gives a range of one-half to two-thirds the average spacing.

#### Methods of closure

There is general agreement that the upper end of a tile drain should be closed as it is obvious that some method, Fig. 1, must be employed to prevent the newly backfilled material and fine silt from washing down into the drain and plugging it up as shown in Figs. 2 and 3. Elliott (13) and Ayres and Scoates (2) do not mention any particular material. Many authors suggest a flat brick or a stone which may be flat or round (19) (80) (30) (48) (72). Etcheverry (14) and Pickels (52) suggest that broken



STONE



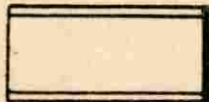
CONCRETE PLUG



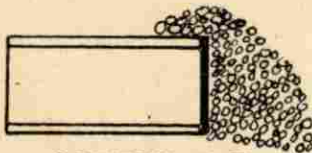
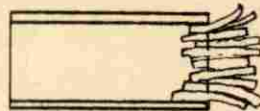
FORMED TILE



SPECIAL PLUG



PLATE

FLAT  
STONE OR BRICKSCREEN WITH  
GRAVEL FILTER

TILE BATS

Fig. 1. Methods of plugging the upper end of a tile drain.



Fig. 2. Result of using a tile bat to close off the end of a drain.



Fig. 3. End tile filled with sediment.

tile bats may be used but Frevert, et al. (22) points out that it is dangerous to do this unless the tile bats are cemented in place. Adkin (1), Waler (79), and Powers (53) do not discuss this point at all. The Mason City Brick and Tile Company are now manufacturing special units for closing the end of drains. This unit is suitable to any size of tile drain (17). Schroeder (65, p. 191) shows a special formed tile and Ferguson (16) suggests that tile stood on end and half filled with cement make very good ends with care being taken not to mix them with the other tile. King (31) states that gravel has been used around broken tile bats and in the past many old tilers used a piece of board or a sod to cover the end. The Soil Conservation Service suggests that any suitable material may be used for this purpose (73) while Chamberlain (5) sums it all up by stating the

. . . upper end of laterals should be stopped up  
 . . . , completely shutting out the dirt and the  
 undue entrance of water. Water should enter there  
 just as much as at other joints, and no more -  
 that is, enter at the narrow crack.

## EQUIPMENT

## Electrical Potential Tank

The laboratory equipment used in this study was essentially the same as that developed by Frevert (20) and Dutz (12). In order to consolidate this information and add to it the equipment and procedure will be detailed here. A rectangular steel tank 10 feet long, 6 feet wide, and 3 feet deep was used. This tank was brushed and cleaned and the interior then sprayed with two coats of Ajax black air drying insulating varnish V61B5, thinned with 20% VM & P naphtha. After use, the tank developed rust spots and it was necessary to touch these spots up by hand. A 24 gauge, 96 inch by 36 inch copper sheet was placed on the bottom of the tank. The remainder of the bottom was covered with copper screen. Lead wires were attached by means of steel screws as solder decomposed and left a scum on the water. Brass screws should be used in the future as the steel screws rusted badly. The screen was difficult to clean but all the conductors were cleaned with a weak solution of muriatic acid.

The recording device as used and described by Frevert (12) was modified so the probe and carriage would move

freely and not stick. A new vertical slide was made and the sheet metal base was changed to a steel base with a ball bearing movement. This improvement aided greatly in securing more accuracy on readings taken near the bottom of the tank and close to the models. Two new probes were made, one for the 2:1 model scale and the other for the 6:1 model scale. These only varied in length. The probes were made of 3/16-inch bronze welding rod. The lower end was filed to a point and a right angle bend was made a distance of 3 inches from the end. The probes were then coated with insulating varnish and after drying were wrapped with black electrical scotch tape. This eliminated the difficulties of leaks that troubled Dutz (12, p. 43). The recorder moved on an aluminum angle-shaped track which was placed across the top of the steel tank.

The electrical apparatus was similar to that used by Dutz (12). A 115-volt to 10-volt 60-cycle bell transformer which was designed for intermittent service was used but due to the high impedance it was feared it might burn out due to the excessive heat. This was replaced by a 110-volt to 6-volt 60-cycle transformer suited to continuous service. This proved satisfactory although the actual potential was only 2 volts due to the high internal resistance in the circuit. The 60-cycle alternating current



was used to prevent polarization and to reduce the amount of coating that forms on the smaller electrodes which increases the resistance. This was reported by Schwab (66). The lower cycles are more difficult to balance than the higher. The wiring diagram is shown in Fig. 4. Two 4000-ohm plug type resistance blocks were used to set the correct ratios desired and a model 302 Electronic Voltmeter, made by Ballantine Laboratories Inc., Boonton, N. J., was used to indicate the null point where the circuit was balanced. It is not necessary to measure both  $E$  and  $I$  in Ohm's law but just the ratio  $\frac{E}{I}$  between these quantities,  $R$ . The resistance readings were taken by using a 400-ohm plug type resistance block in conjunction with the two 4000-ohm resistance blocks and forming a Wheatstone bridge circuit. The vacuum tube voltmeter was used to determine when the circuit was balanced. A Triplet model 630 A Volt-Ohm-Ammeter was used to check the voltages and resistances.

### Models

The choosing of a model scale which is proportional to the boundary conditions or size of tank is very important. If properly constructed, the wall effect will give a superposition of an infinite number of images which is of use in solving certain flow problems. The model should be as large as possible as the errors of small deviations

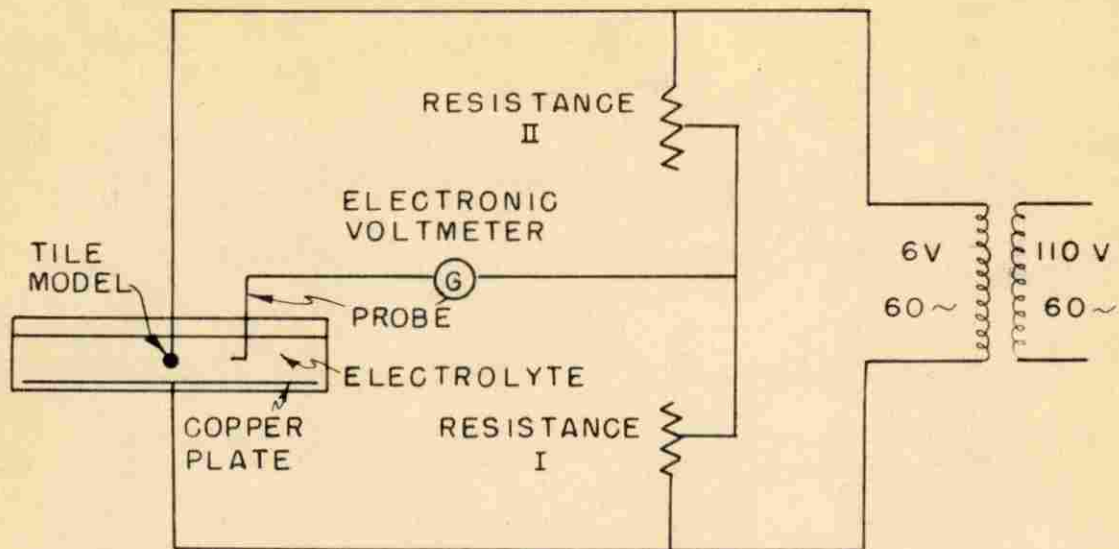


Fig. 4. Wiring diagram for determination of equipotential lines.

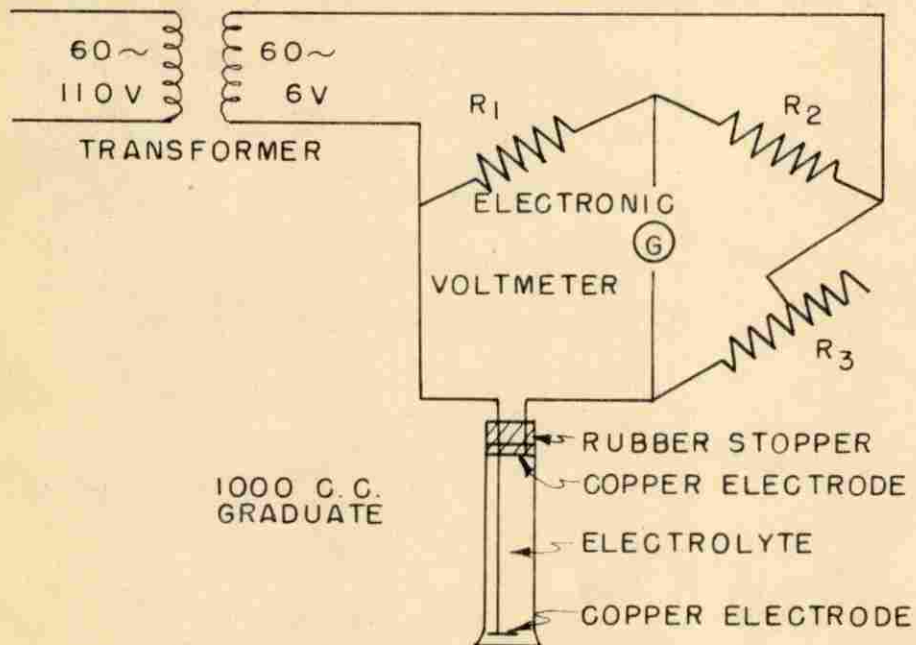


Fig. 5. Wiring diagram for determination of conductivity of electrolyte.



are greatly magnified. For the same reason the electrodes or probes for measuring should be quite small. Preliminary tests with a 2:1 model scale proved unsatisfactory because of the difficulty in duplicating the spacing for the different model scales that were used. It was decided to use a 6:1 model scale so that the outside drain diameter of a 5-inch tile, which is 6 inches, would correspond to a 1-inch model.

The 1-inch model drains were attached to a support board a distance of 8 inches from the center of the drain to the bottom of the tank. This is the same as placing the drain 4 feet deep in the ground. The tank was filled with tap water to a depth of 16 inches which corresponded to a depth to the impermeable layer of 8 feet with the drains midway between the ground surface and the impermeable layer. The insulated sides of the tank represented the midpoint of the drain spacing. The distance was 72 inches which corresponded to a spacing of 36 feet.

Figs. 6, 7, and 8 show illustrations and photographs of the various models that were used in this study. The model that was used to represent the totally open or unlined drain for the direct analogue is shown in Fig. 6(c), and Fig. 8(2). This was a type K copper pipe 5 feet long and 1 inch in diameter with a copper disc over one end.

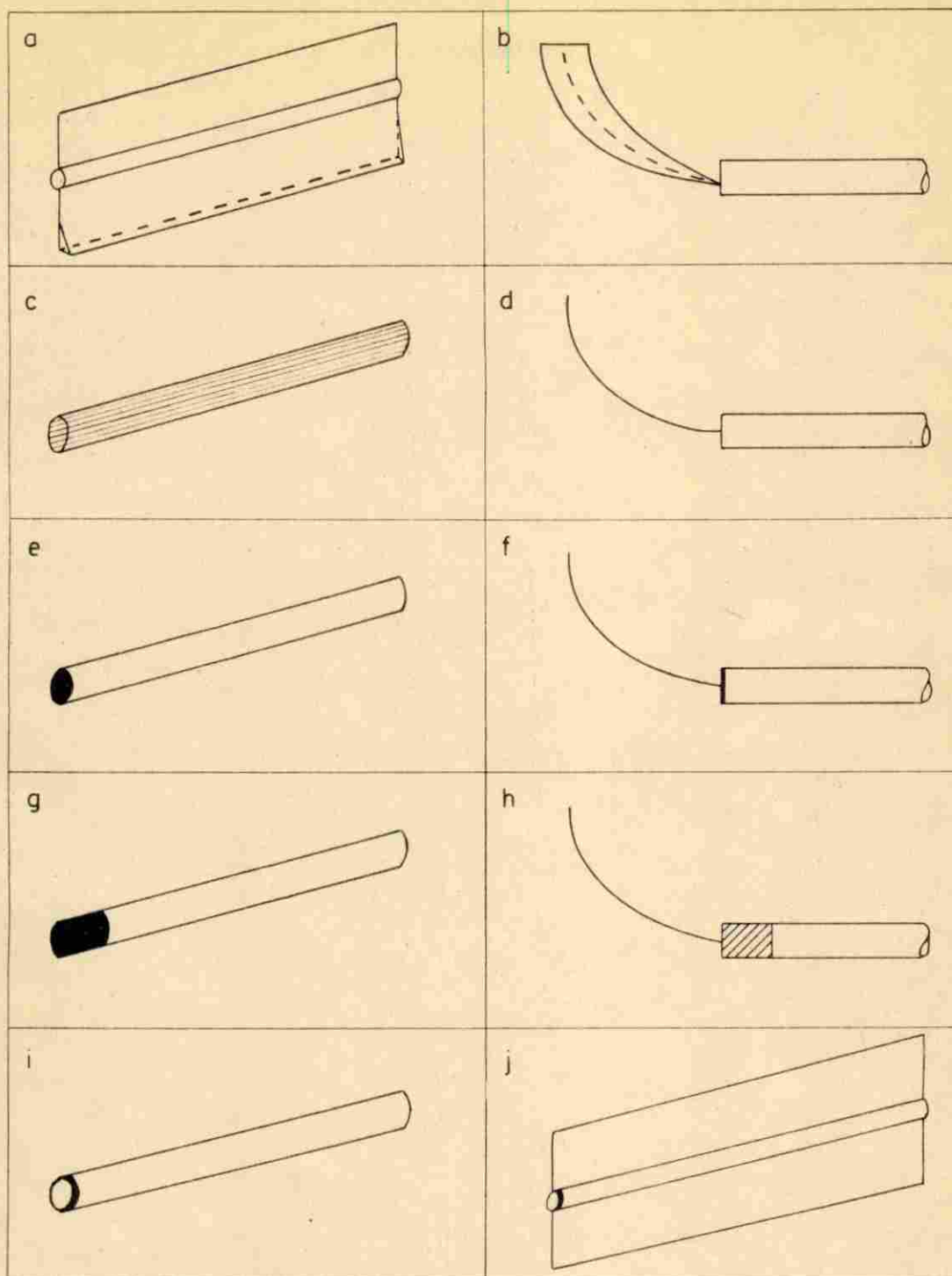


Fig. 6.\* Drawing of drain models used with the electric analogue.

\*Inverted models a, b, d, f, h, j are for streamlines.  
Direct models c, e, g, i are for equipotential lines.

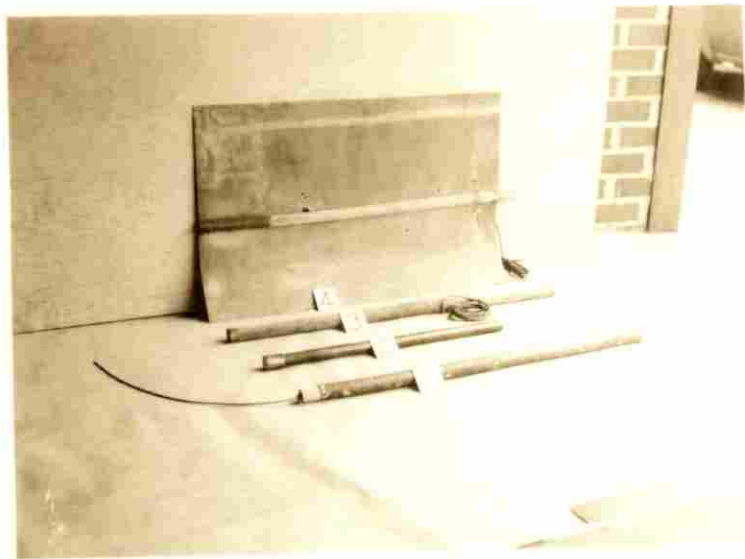


Fig. 7. Models of drain tubes.



Fig. 8. Models of drain tubes for totally open drain.

A lead wire was soldered to the pipe and the pipe placed on the support board with a plastic tube holding up the other end.

The model representing the gravel packed end (Fig. 1) in the direct analogue was a solid wooden dowel 5 feet long and 1 inch in diameter which was insulated with varnish. A 1-inch copper cylinder, 1-1/2 inches long, was fitted to the dowel and the lead wires attached to the copper conductor. This is shown in Figs. 7(2) and 6(g).

The model representing a closed drain with an open end was a solid wooden dowel 5 feet long with a copper disc fitted to the end as shown in Figs. 7(3) and 6(e). The lead wire was attached to the disc and the dowel painted with insulating varnish.

The model used to represent the 1/8-inch crack at the end of the tile line was the same as that used for the open end test with the exception that the copper disc was painted with insulating varnish except for a ring at the outer edge of the disc. This model was very difficult to work with due to its small size but the results compared favorably with tests on larger model scales.

In order to invert the analogue it was necessary to make all the previous conductors insulators and the insulators now become conductors, or more generally the known

boundary streamsurfaces should be made equipotential surfaces and the normal equipotential boundaries be made insulated or free surfaces. This means that the insulated sides of the tank should be covered with copper as well as the bottom. This would give five conducting surfaces which should all be joined together. For the inverted analogue of the totally open drain, in a plane normal to the axis of the drain, a model was made using a solid wooden dowel in which two slots were milled and two sheets of copper fitted as illustrated in Figs. 6(a) and 7(4). The wooden dowel was insulated and wires attached to each sheet of copper. Each sheet was at a different potential. A slight curvature was placed on the lower electrode or boundary. The reason for this curvature is that the copper boundary has a finite thickness and so would not represent the 100 per cent streamsurface but some value less than 100 which would have a curved surface.

For the inverted analogue of the unlined drain completely pervious along its whole length (which would represent a drain tube embedded in gravel) in a plane vertical with the axis of the drain, a solid wooden dowel was insulated and a 3/32-inch welding rod bent to fit orthogonal to the equipotential lines obtained from the direct analogue was used to represent this case. This welding rod was



fitted into the end of the dowel as illustrated in Figs. 6(d) and 8(1). The other end of the welding rod was held above the water surface. A cone, Figs. 6(b) and 8(3), was also used in place of the bronze welding rod. The reason for this is that the welding rod represents the zero stream-line boundary for purposes of determining the actual stream-lines and because it has a finite thickness, the stream-lines having no thickness, it does not represent the zero streamline but rather some small value greater than zero. This means the welding rod should be actually cone-shaped. The diameter of the cone at the water surface is difficult to estimate.

The inverted models for the other cases considered are shown in the illustrations. The inverted model for the case of the 1/8-inch crack is shown in Fig. 6(j), that for the open end drain is shown in Fig. 6(f) and that for the gravel packed end is shown in Figs. 6(h) and 7(1).

## PROCEDURE

## Model Study

The model experiment was carried out in a large rectangular steel tank 10 feet long, 6 feet wide, and 3 feet deep. The interior of the tank was brushed, cleaned and sprayed with two coats of a quick drying insulating varnish. Later in the experiment the tank developed rust spots in several places due to uneven application of the varnish. The tank was drained and these spots touched up by hand. A 24-gauge sheet of copper plate 96 inches by 36 inches was placed on the bottom of the tank. A copper screen was used to cover the area not covered by the sheet. The chief disadvantage in using screen was the difficulty in cleaning it. There was no measurable difference in resistance in using the screen or copper sheet. The lead wires were attached to the screens and copper plate. In the future it is advisable to use brass screws for this purpose. The tank was then filled with tap water to a depth of 16 inches.

The temperature of the water after the tank was filled to the required depth was 56° F. A conductivity test, Fig. 9, was made on a sample of the electrolyte. The sample of

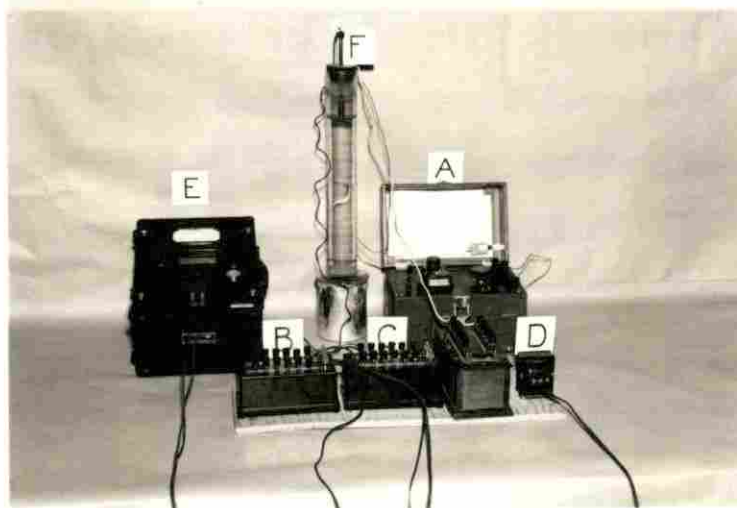


Fig. 9. Apparatus used to determine conductivity of electrolyte.

- A - potentiometer
- B - 4000 ohm resistance block
- C - 4000 and 400 ohm resistance block
- D - 110 to 6 volt transformer
- E - electronic voltmeter
- F - conductivity flask



water was placed in a 1000 cc. graduate and a copper electrode 2.198 inches in diameter was placed, with a lead wire attached, on the bottom of the graduate. Another copper electrode was placed a distance of 12 inches from the bottom electrode. The wiring diagram is shown in Fig. 5. The resistance between the copper electrodes was then measured and the temperature taken simultaneously. This was repeated until the water or electrolyte reached a constant temperature. Difficulties were encountered in measuring this resistance because an unreliable Wheatstone bridge was used. The values obtained from it did not agree with those measured by the Model 630 A Ohmmeter. The Wheatstone bridge was replaced by three resistance blocks set up in a bridge circuit. Each resistance block checked out, but the same differences in readings were encountered as before. The set up was checked with a standard variable resistor (Central Scientific Company) and it checked out. The variation in resistance readings across the electrolyte using the bridge and the Triplet Ohmmeter must have been due to the high internal voltage of the meter.

An electrolyte obeys Ohm's law except for very high voltages and high frequency currents (81). Thus for an applied potential, the voltage remaining constant, the current will vary inversely with the resistance  $R$  of the

electrolyte. The conductivity of an electrolyte, the conductance  $\underline{C}$ , is taken as  $1/R$  and is expressed in mhos.

$$C = 1/R = k A/L$$

where  $\underline{C}$  is the conductance in mhos,  $\underline{R}$  the resistance in ohms,  $\underline{k}$  is the specific conductivity in mhos per inch cubed,  $\underline{A}$  is the area of the conductor in square inches, and  $\underline{L}$  is the length of the conductor in inches.

$$k = L/RA = 12/R \times 3.77 = 3.18/R$$

The temperature readings were taken using a 20-gauge wire copper-constantan thermocouple. The readings were taken on a Leeds and Northrup Potentiometer Indicator which was equipped with an internal cold junction compensating device. The potentiometer was calibrated to read for iron-constantan so a conversion chart was prepared by using a  $1/2^{\circ}$  F precision thermometer to standardize the potentiometer. This is shown in Fig. 10. A regression line was made for the variation of specific conductivity with temperature, Fig. 11. The increase in resistance was due to the coating of the small electrodes. The experiment was conducted in a thermostatically controlled room. In four days the electrolyte had risen to and remained constant at  $70^{\circ}$  F.

In general the flow of water through earth masses is three-dimensional; however, this situation is too complicated

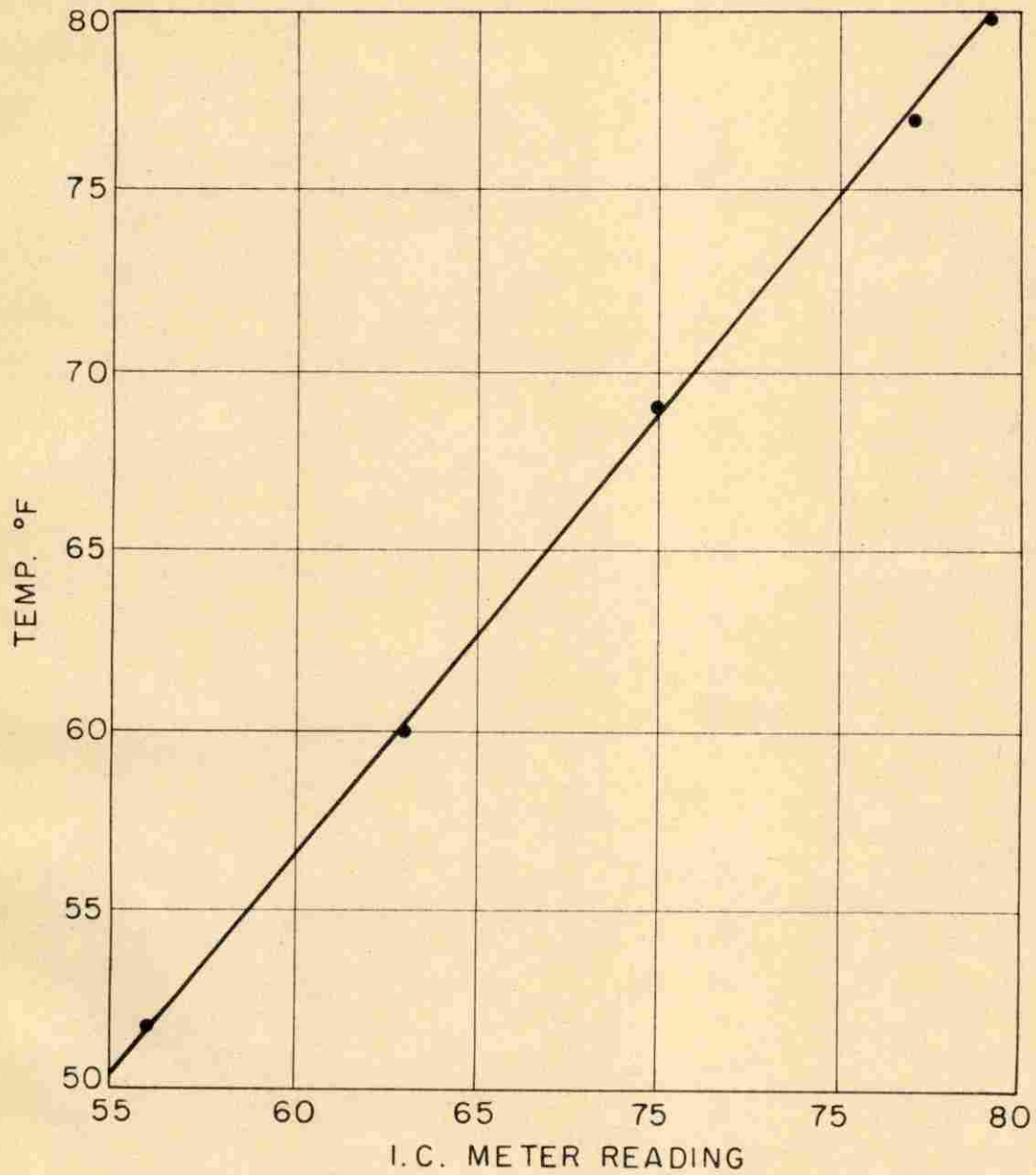


Fig. 10. Temperature conversion graph for iron-constantan thermocouples.

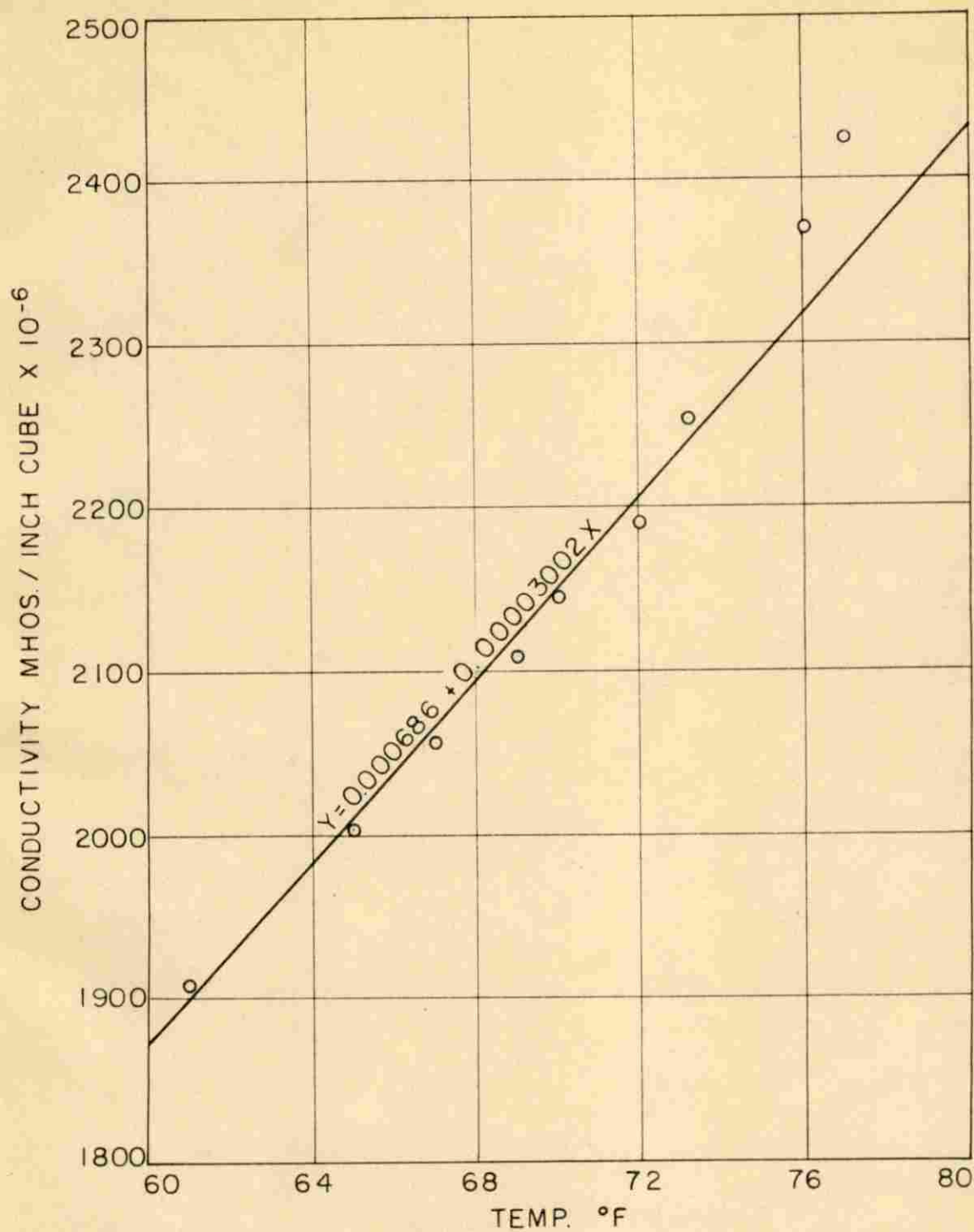


Fig. 11. Effect of water temperature on conductivity of electrolyte.

for practical analysis and the fundamentals of flow can best be represented by using the simple two-dimensional case. The significant character of two-dimensional flow is that all the features of the motion may be observed in a single plane, the motion being identically the same in parallel planes. The case considered here is three-dimensional but it is best represented by considering a two-dimensional system in two planes as shown in Fig. 17.

The equipment which has been described previously was assembled on the tank as shown in Fig. 12. The wiring diagram used for this experiment is illustrated in Fig. 4. The first test model was placed in position. This was the totally open or unlined drain in the direct analogue. The procedure was to trace the 0.1, 0.2, to 0.95 equipotential lines by first setting the correct bridge ratio, taken from Table 1, on the resistance blocks and then moving the recorder as illustrated in Fig. 13 until the electronic voltmeter registers a null point. A mark is made on the paper at this point. The bridge ratio is changed and the recorder moved until a further point is made. This is done by the cross section method. It is much speedier to complete each equipotential line before changing the bridge ratio. When a series of points are made, all points with the same potential are joined by a smooth curve. The model



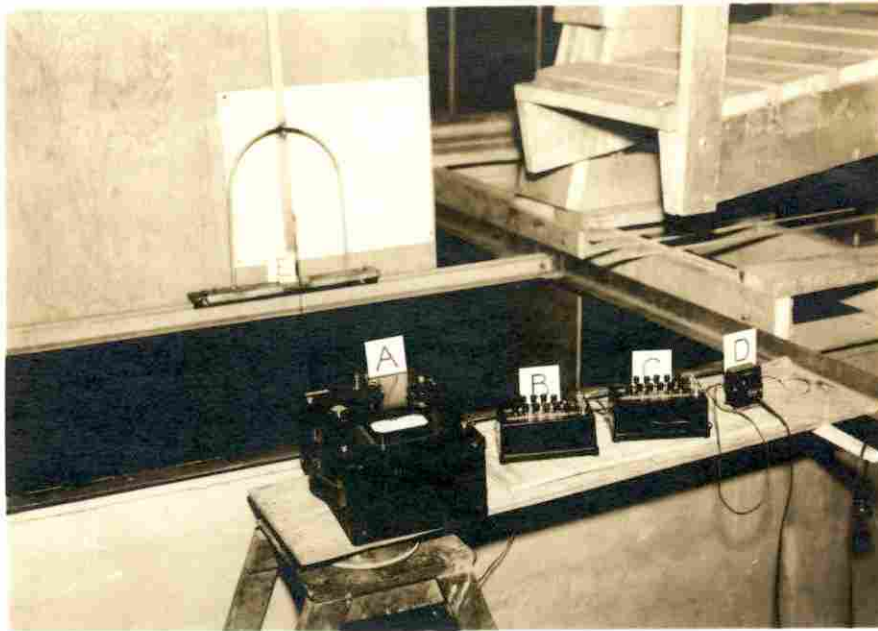


Fig. 12. Arrangement of equipment for model studies.

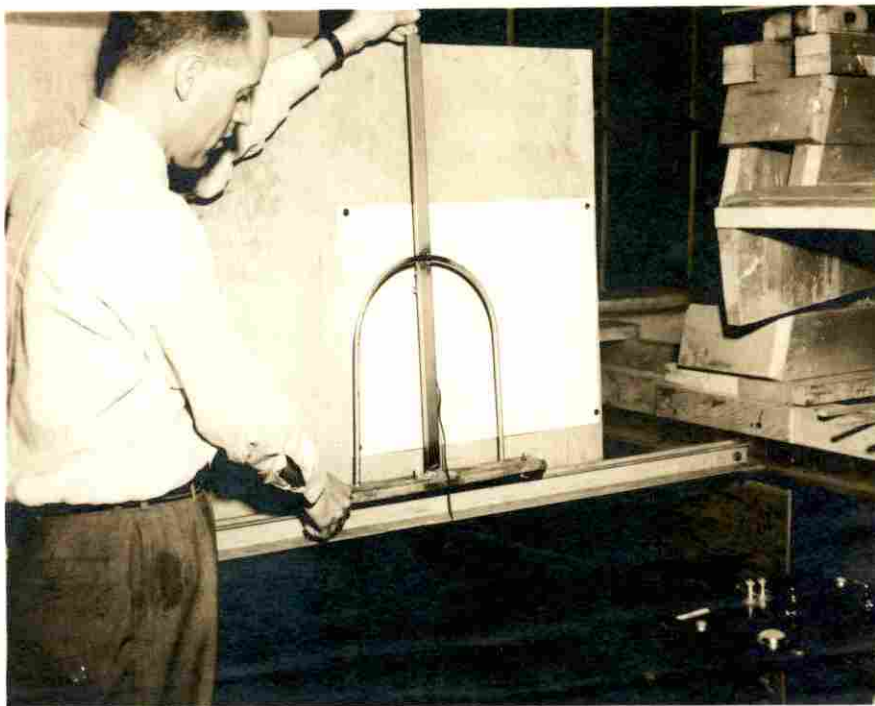


Fig. 13. Recorder and method of operation.

Table 1. Resistance ratios for plotting equipotential lines and streamlines.\*

Equipotential and streamline value	Resistance block I	Resistance block II
Per cent	Ohms	Ohms
10	900	100
20	800	200
30	700	300
40	600	400
50	500	500
60	400	600
70	300	700
80	200	800
90	100	900
95	50	950

\*Calculated from p. 2196 of reference (27).



was changed and a new set of curves traced.

A series of cross sections were made on the totally open drain in the XY plane and all the points of equal potential were joined to give one family of curves. A family of equipotential lines were traced in the XZ plane, and in the YZ plane at  $X = 8$ . The boundaries were then changed and also the models in order to invert the analogue. A series of streamlines were traced in the same manner as the equipotential lines. The equipotential lines and the streamlines were then superimposed to form the flow net.

In a similar manner, flow nets were developed for the case of the 1/8-inch crack, the open end drain, and the gravel packed end.

### Field Study

In drainage investigations the theoretical solutions and solutions derived from model studies must be tested in the field where the results of these studies must eventually be put into practice. This study was to find how the water table at the upper end of a tile drain was affected by various practices. A limited study was made on the various methods now employed in closing off the upper end of lateral drains and to evaluate two of these methods.

Accompanying this study was an experiment to determine the radius of influence at the upper end of a tile drain so that some guide could be used for the laying out of plots in experimental work to make the most efficient use of the space available. From this same experiment an estimate of the distance a tile drain might be expected to adequately drain the land off the end of the drain was made.

Agricultural Engineering Research Farm, Ames

The Agricultural Engineering Research Farm at Ames was selected as a site to determine the effect the upper end of a tile drain has upon the water table around the end of the line and at some point down grade from the end. The site selected was line two <sup>( $\frac{1}{8}$ " crack)</sup> of the crack spacing experiment which has been described by Dutz (12). The choice was based on the uniformity of the soil and the distance from other drains. This line consisted of 80 feet of 5-inch tile, which had a steel plate over the end, and 20 feet of steel pipe which outletted into a sump.

The end of the tile line was found with a probe and the layout of the water table pipe was made. This is shown in Fig. 14. Above the end of the tile line pairs of water

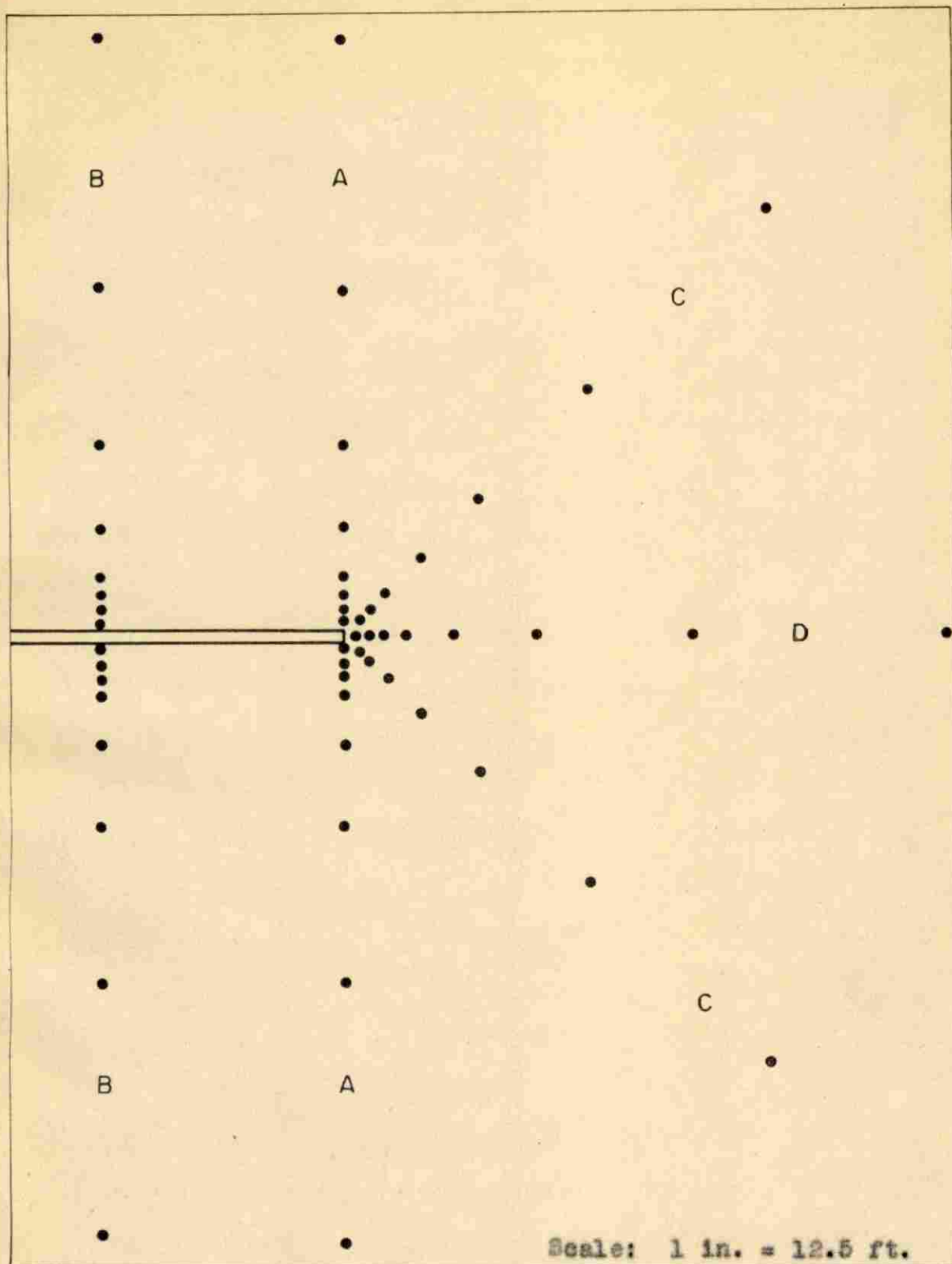


Fig. 14. Layout of water table pipe at the Agricultural Engineering Research Farm, Ames.

table pipes were set out logarithmically in a line projected from the axis of the drain. This is line D. Line C was laid out in the same manner at 45 degrees to this first line and line A was laid out normal to the axis of the drain at the upper end of the drain. Water table pipes were placed in pairs at each location which was 1 foot, 1.8 feet, 3.2 feet, 5.2 feet, 9.1 feet, 16.2 feet, 29 feet, and 50 feet from the origin which was directly over the end of the tile line. At a distance of 20 feet down grade from this origin another line (B) was set out normal to the axis of the drain. This gave a setup with two replicates, except for line D.

The water table pipe were installed by using a 15/16-inch Irwin car bit brazed to a 4-foot extension and powered by a 5/8-inch electric drill. This worked very satisfactorily and saved considerable labor and time, 40 water table pipe being installed in two hours. The lower ends of the 5-foot pipe near the drain were augered by hand.

The spring was very dry and no high water table developed on the location. A heavy stand of brome grass also tended to deplete the available water. The water table came up to 14 inches from the surface so a pump was installed and the water pumped from the sump flooding the area over the end of the tile. This water came from the

other drains in the crack spacing experiment. The number two line was plugged until the soil was thoroughly saturated. As the water table receded a series of water table levels were taken with a reel type recorder as described by Luthin (40).

Clarion-Webster Soil Association  
Experimental Farm, Kanawha

At the Clarion-Webster farm at Kanawha an experiment was set out to determine the effect on the water table of two methods of closing off the upper end of a tile drain. Six drains were selected and with the two treatments this gave three replicates. The treatments were randomized. The experiment layout is illustrated in Fig. 15.

On laterals A-5.16, A-5.14, and A-5.12 a hole was dug down to the drain and the present plugs, which were tile bats, were removed and a copper plate which was drilled with 30 1/4-inch holes was placed over the end. Two cubic feet of sand was then placed next to the copper plate and extending for one and one half feet off the end of the drain. This is similar to Fig. 1 with the gravel filter. The excavations were then filled in and tamped well to maintain the present density as much as possible.

On laterals A-5.15, A-5.13, and A-5.11 the tile bats

○ COPPER PLATE WITH SAND OVER END  
 X END PLUGGED WITH CONCRETE

SCALE: 1 INCH = 100 FEET

(WATER TABLE PIPE NOT TO SCALE)

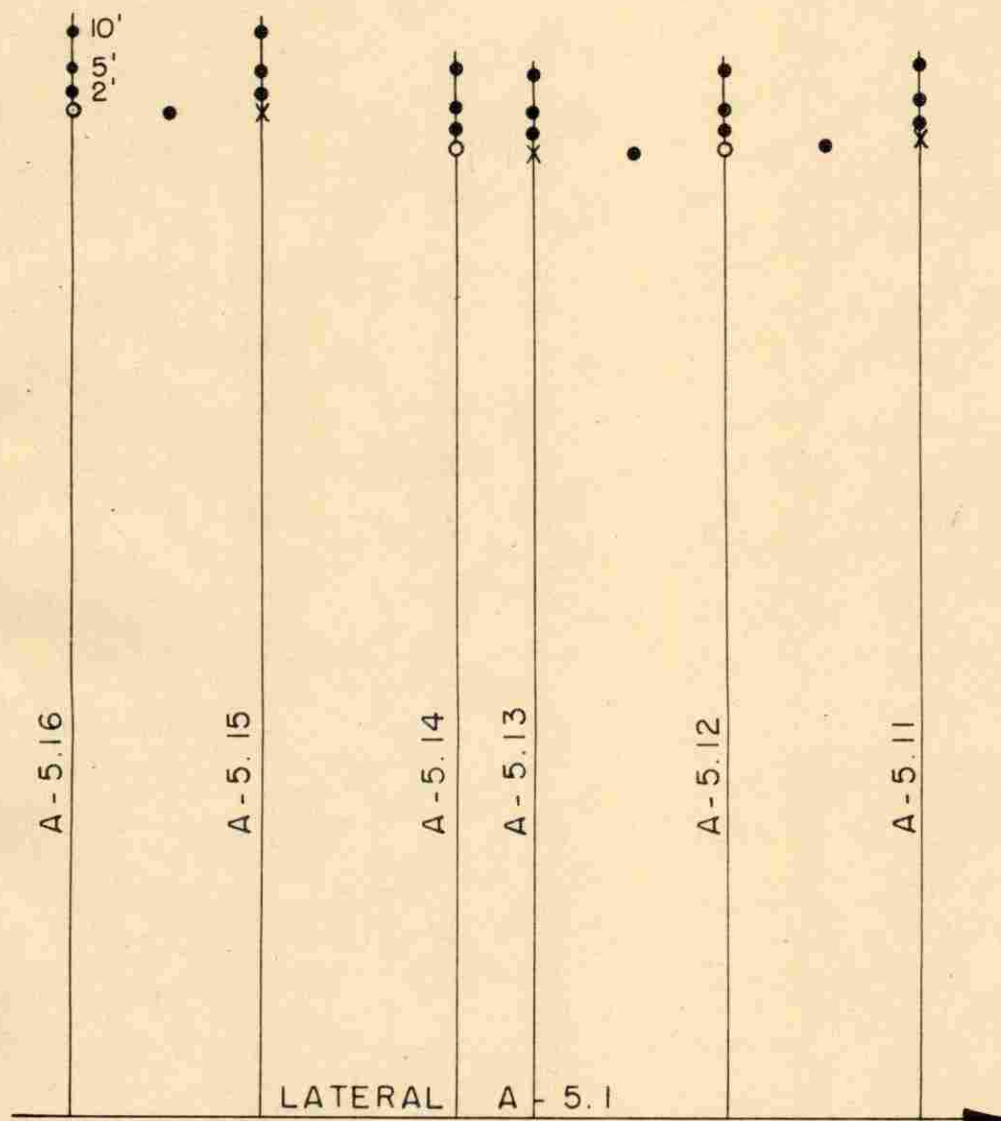


Fig. 15. Layout of water table pipe at the Clarion-Webster Soil Association Experimental Farm, Kanawha.



were removed and the ends filled with concrete. Pairs of water table pipe were placed in the ground at a distance of 2 feet, 5 feet, and 10 feet from the ends of the lateral drains. Water table pipe were also placed midway between A-5.16 and A-5.15, A-5.13 and A-5.12, and also between A-5.12 and A-5.11 and in a line even with the end of the drains. These laterals are 500 feet long.

#### Howard County Experimental Farm

The Howard County Experimental Farm was selected on which to place an experiment to determine the water table shape around the end of the tile line and also the shape of the water table at a point midway between two adjacent drains. The layout of the experiment is shown in Fig. 16. The upper end of lateral line 5.32 was found with a probe and water table pipe were placed in pairs a distance of 2, 5 1/4, 10, and 15 3/4 feet from the end of the line. This was line D. Line A was placed normal to the axis of the drain at the upper end and pipes were installed in pairs at 5-1/4, 15-3/4, and 42 feet from the drain. From the 42-foot mark line E was set out parallel to the drain. The pipe in this line were placed 6 feet apart for 18 feet in both directions from the original pipe. A single pipe



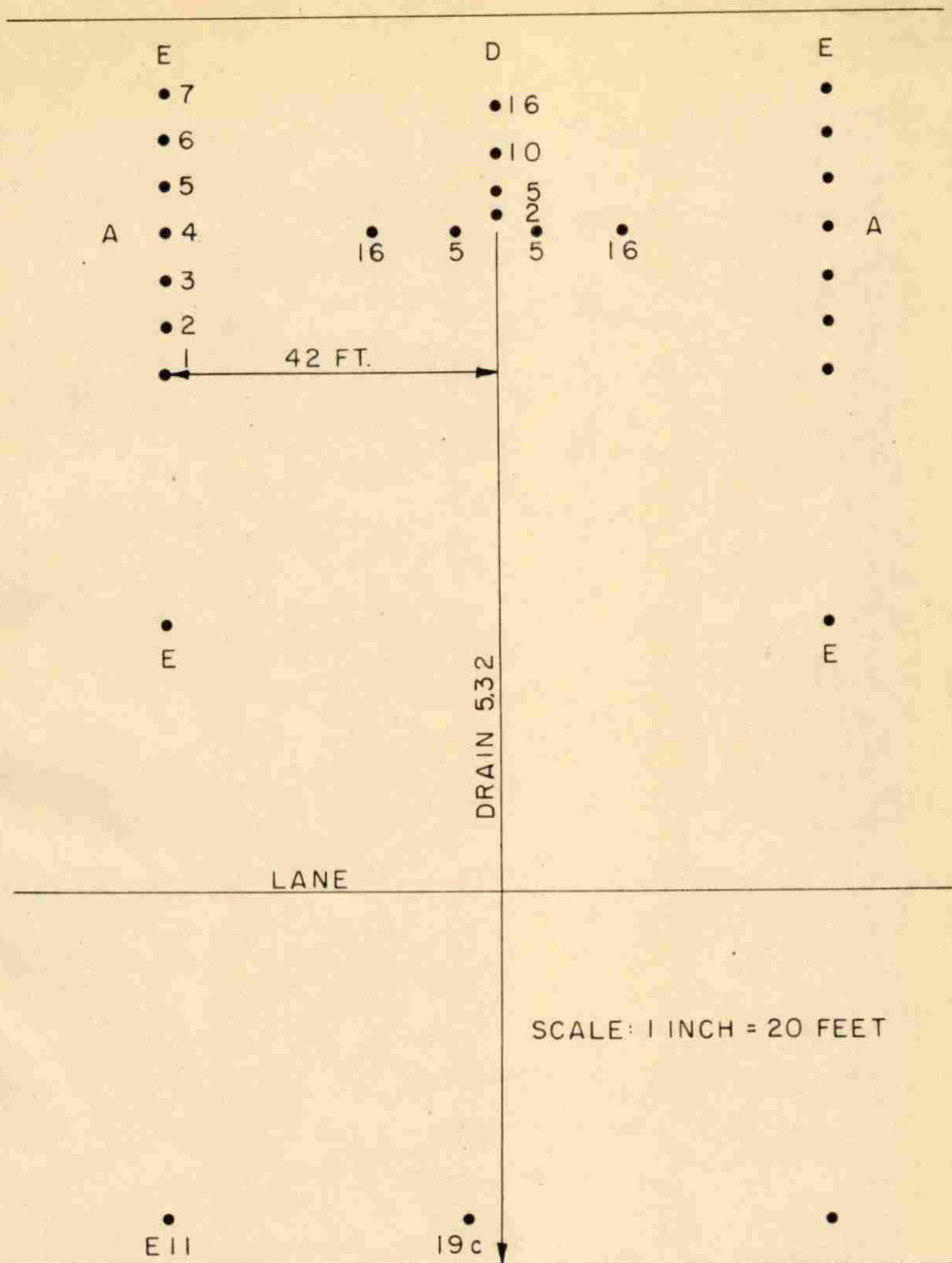


Fig. 16. Layout of water table pipe at the Howard County Experimental Farm.

was installed 50 feet down grade from the original pipe in this line. Water table pipe installed in another experiment north of the lane was also tied into this experiment.

## RESULTS

## Model Study

A flow net facilitates the study of the gravitational flow of seepage or drain water through the soil. It consists of two families of curves which bear a fixed relationship to each other and represents the pattern of the flow and the dissipation of the head causing the flow. One family of curves is called equipotential lines and each line passes through points of equal head and each space between adjacent equipotential lines represents an equal drop in head, or the force causing the flow.

The other family of curves is known as flow lines or streamlines and the direction of the fluid at any point coincides with the tangent at that point of these curves. In a true flow net under uniform flow the figures are true squares, however, under steady nonuniform flow as here they form curvilinear squares. The distance between the streamlines is inversely proportional to the velocity. Streamlines in an unsaturated area are vertical (25) and do not change shape with a change of scale. The results may be interpreted at any scale provided the drain diameter and boundaries are in scale. The quantity of seepage

flow between adjacent pairs of flow lines is constant. Although there are an infinite number of flow lines it is convenient to choose only a limited number so that the quantity of flow between each pair is equal.

The determination of flow nets and water table shape around the upper end of a subsurface drain is a three-dimensional problem and as such is very difficult to analyse. The approach has been to analyse the problem by taking a two-dimensional problem in three planes. Fig. 17 shows the orientation of the planes in relation to the tile. The streamlines in this figure are marked with arrow heads. In the same figure it may be seen that the equipotential lines actually form bullet-shaped equipotential surfaces which the streamlines cut orthogonally.

Fig. 18 is the direct analogue of a totally open drain showing the equipotential lines in the XY plane. It should be noticed that these equipotential lines form exponential curves which converge in passing around the end of the drain and then diverge until they meet an image plane at the end of the tank which in the figure is below section line B. If the drain were long, these equipotentials would approximate parallel lines. At the bottom of Fig. 18 is a plot, at double the scale, of the equipotential lines along the section lines as indicated on the

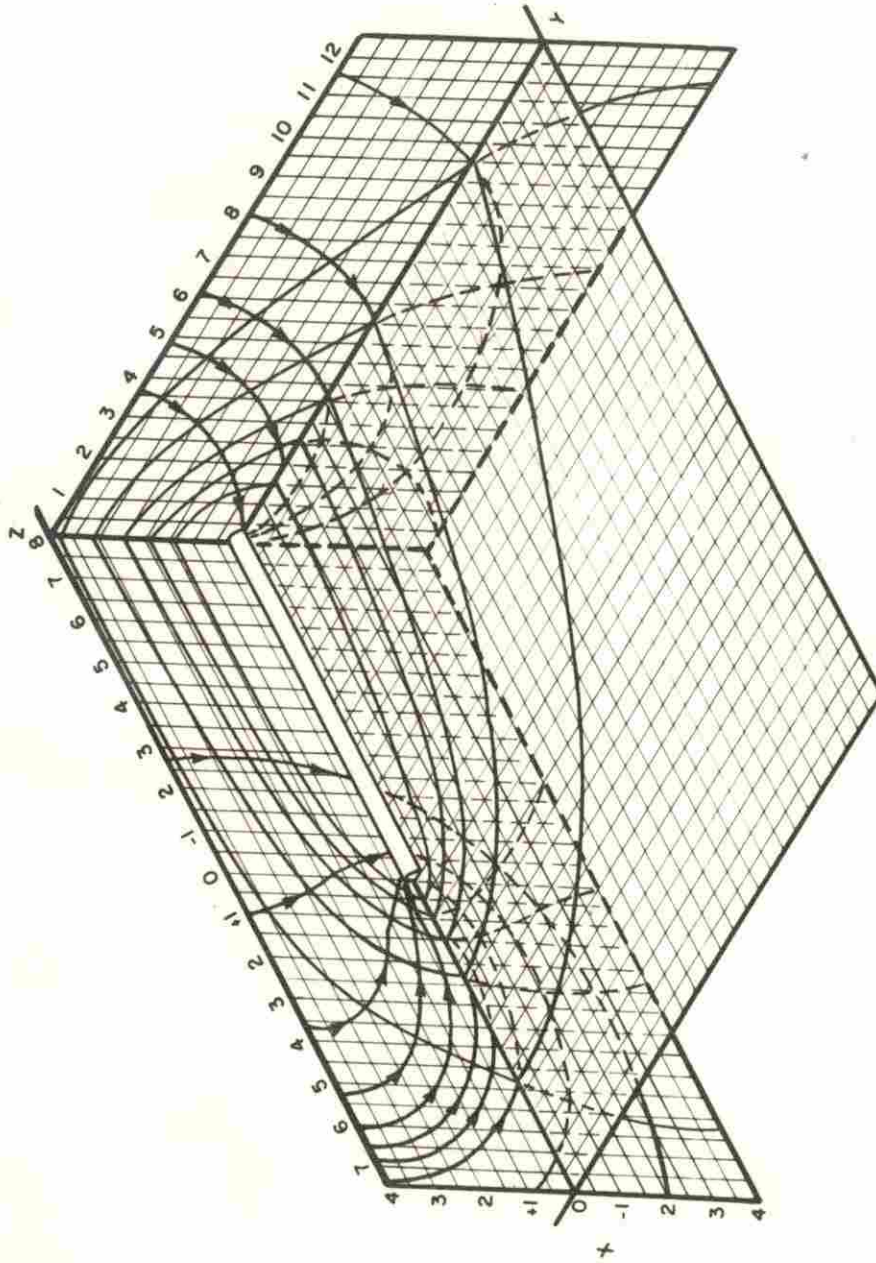


FIG. 17. Drawing showing the orientation of the three planes.



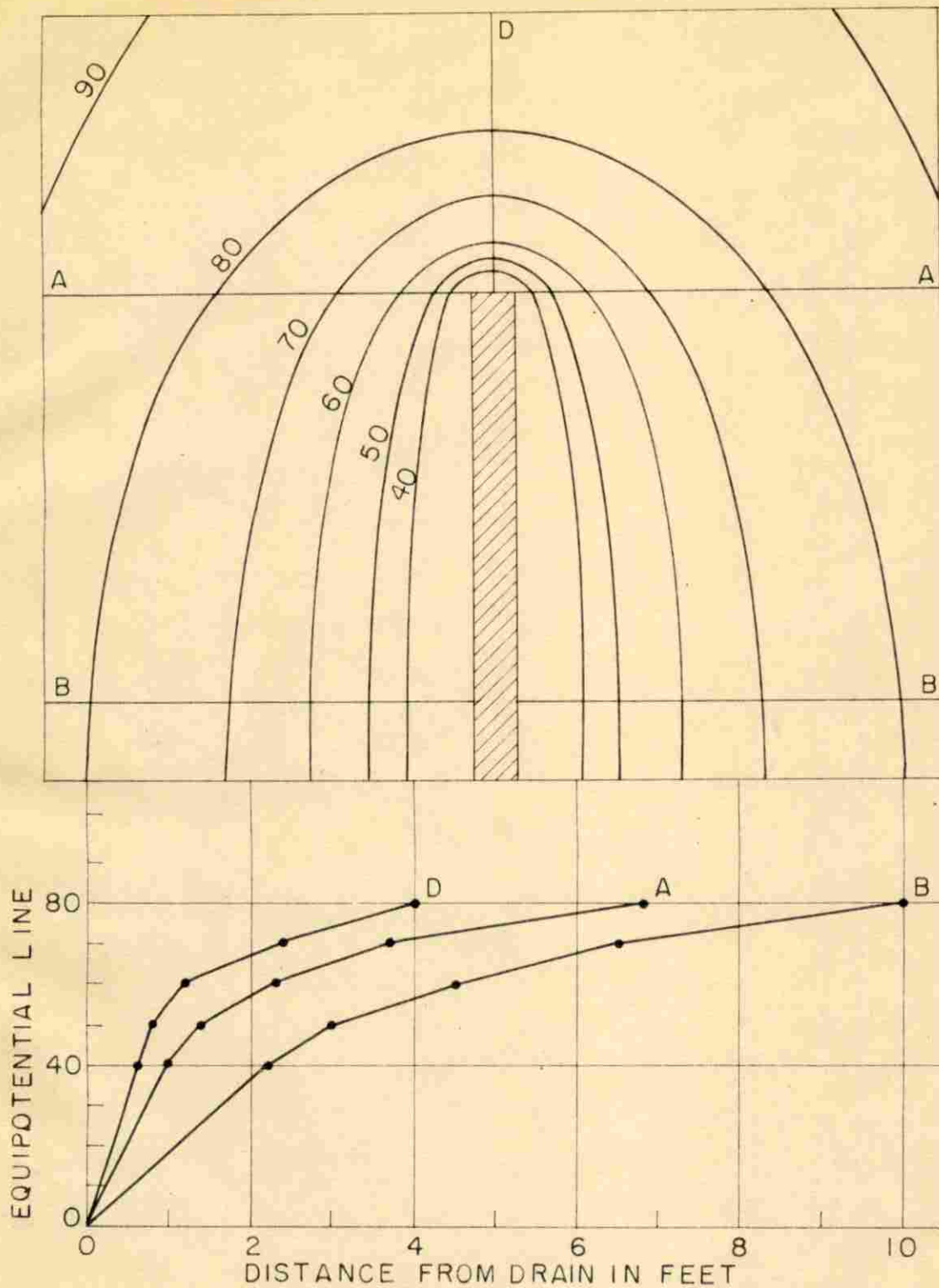


Fig. 18. Equipotential lines in the XY plane around the end of an unlined drain and plotted against distance from the center of the drain along the section lines.

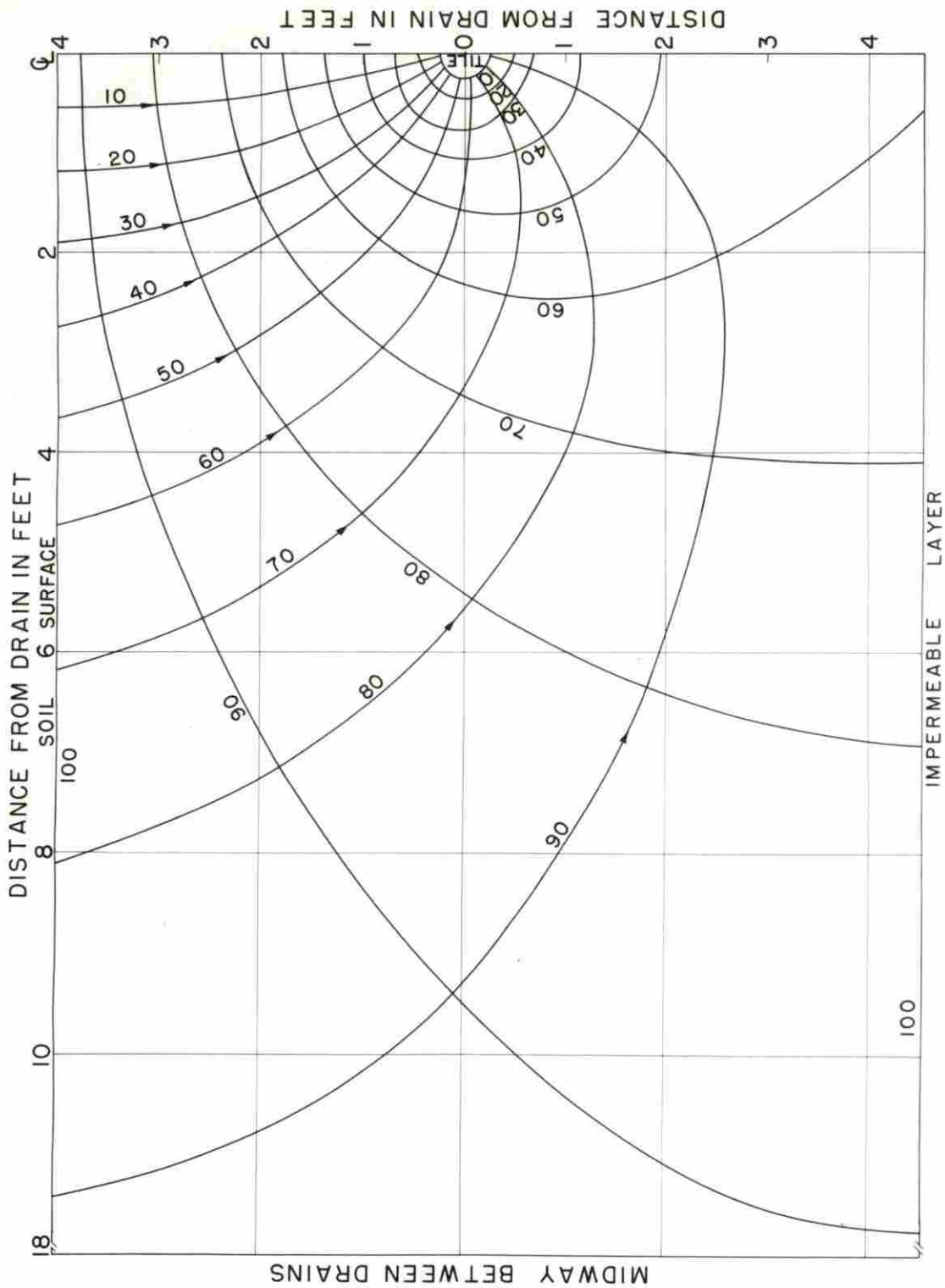
figure. It can be seen that line B is much lower than line A or D which indicates better drainage at B. If piezometers were placed to a depth equal to the horizontal axis of the drain and at positions where the equipotential lines cut the B section line in Fig. 18 then the height to which the water would rise in the piezometer tubes would give an approximation of the water table if there was no deep seepage or artesian pressure and the soil was homogeneous. The figure is really a plot of head loss versus distance from the drain. As the distance from the drain gets smaller the accuracy decreases.

The totally open or unlined drain and the drain with an open end is of academic interest only. In the case of the unlined drain in the YZ plane it may be seen in Fig. 19 that a vertical line through the axis of the drain forms a line of symmetry. The flow net resulted from superimposing streamlines from the inverted analogue upon the equipotential lines obtained from the direct analogue. The unlined drain gives the maximum flows that can be expected and the other cases are based on this assumption. Fig. 20 illustrates the flow net derived for the unlined drain in the XZ plane. This flow net is difficult to analyse. The equipotential lines and streamlines are orthogonal which would indicate that the pattern is correct,



Fig. 19. Flow net of the unlined drain in the YZ plane at  $X = 8$ .

NOTE: In Figs. 18 to 26 equipotential and streamlines follow the actual points determined by the electric analogue. The lines theoretically should intersect orthogonally; deviations are due to experimental error.



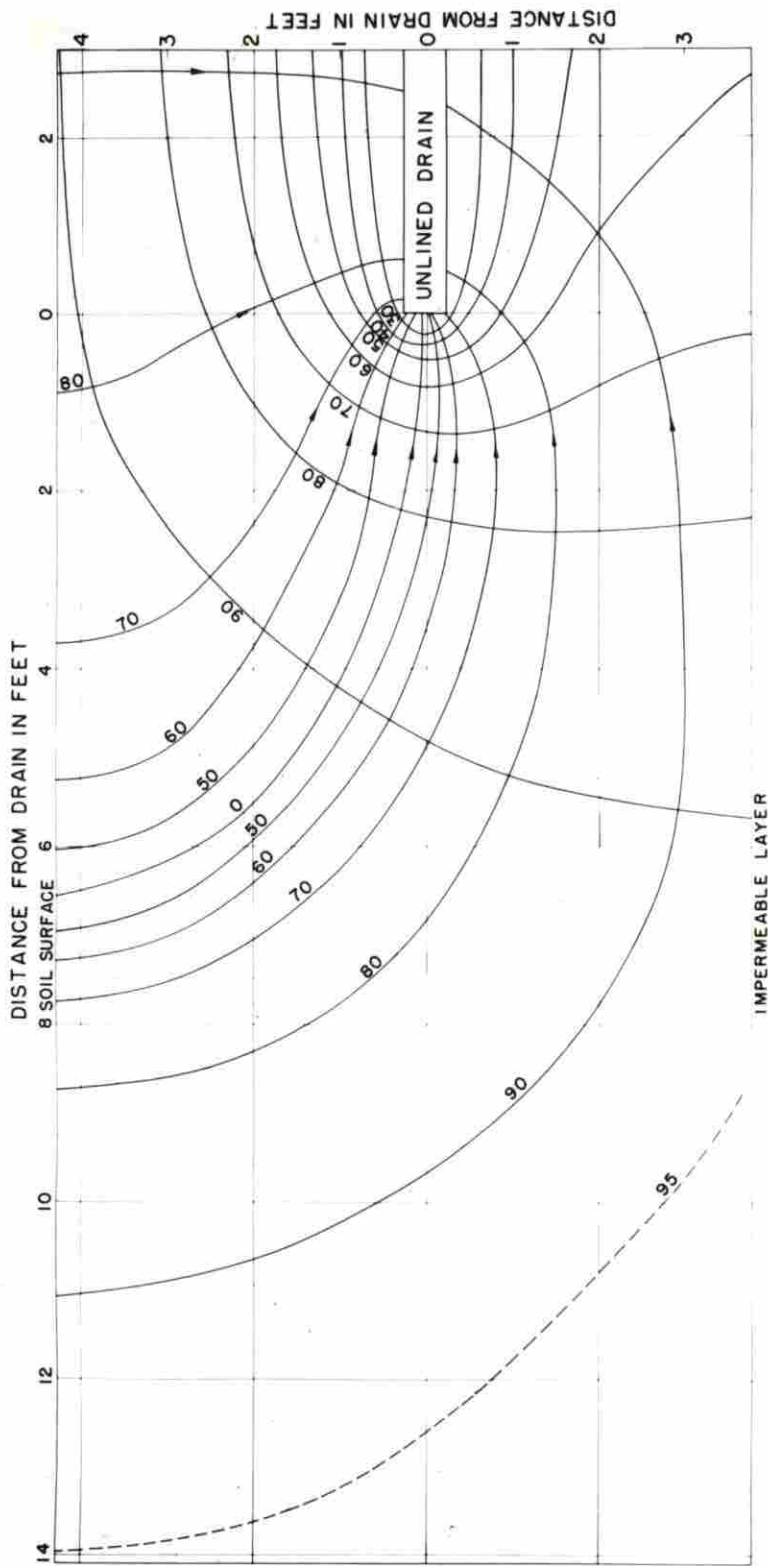


Fig. 20. Flow net of the unlined drain in the xz plane.

however, the flow net indicates that a larger volume of flow would enter the soil surface at a distance of 7 feet from the end of the tile line. This does not agree with observations in the field. At a distance of 7 feet from the drain it is seen that 50 per cent of the total flow should enter the soil in a diameter of 1-foot. This does not appear to be reasonable, however it is difficult to find a fallacy in this flow net analysis. The same type of flow net pattern is shown by Gregg (23, p. 288) and in Low Dams (74, p. 283-284) with no explanation. In Fig. 19 the 50 per cent equipotential line is 18 inches from the drain (off the side at  $X = 8$ ) and in Fig. 20 of the unlined drain off the end the 50 per cent equipotential line is 6 inches from the drain. This would indicate that there is not as great a loss of head off the side of the drain, at a distance of 8 feet back from the end of the tile, as there is off the end of the drain.

Present drainage practices leave a crack at the upper end of a tile drain. In Fig. 21 the flow net representing the YZ plane shows a large reduction in head compared to the case of the totally open drain in Fig. 19. In Fig. 22 the flow net representing the 1/8-inch crack in the XZ plane also shows this tremendous loss of head. Difficulties were encountered in tracing the flow net representing the 1/8-inch

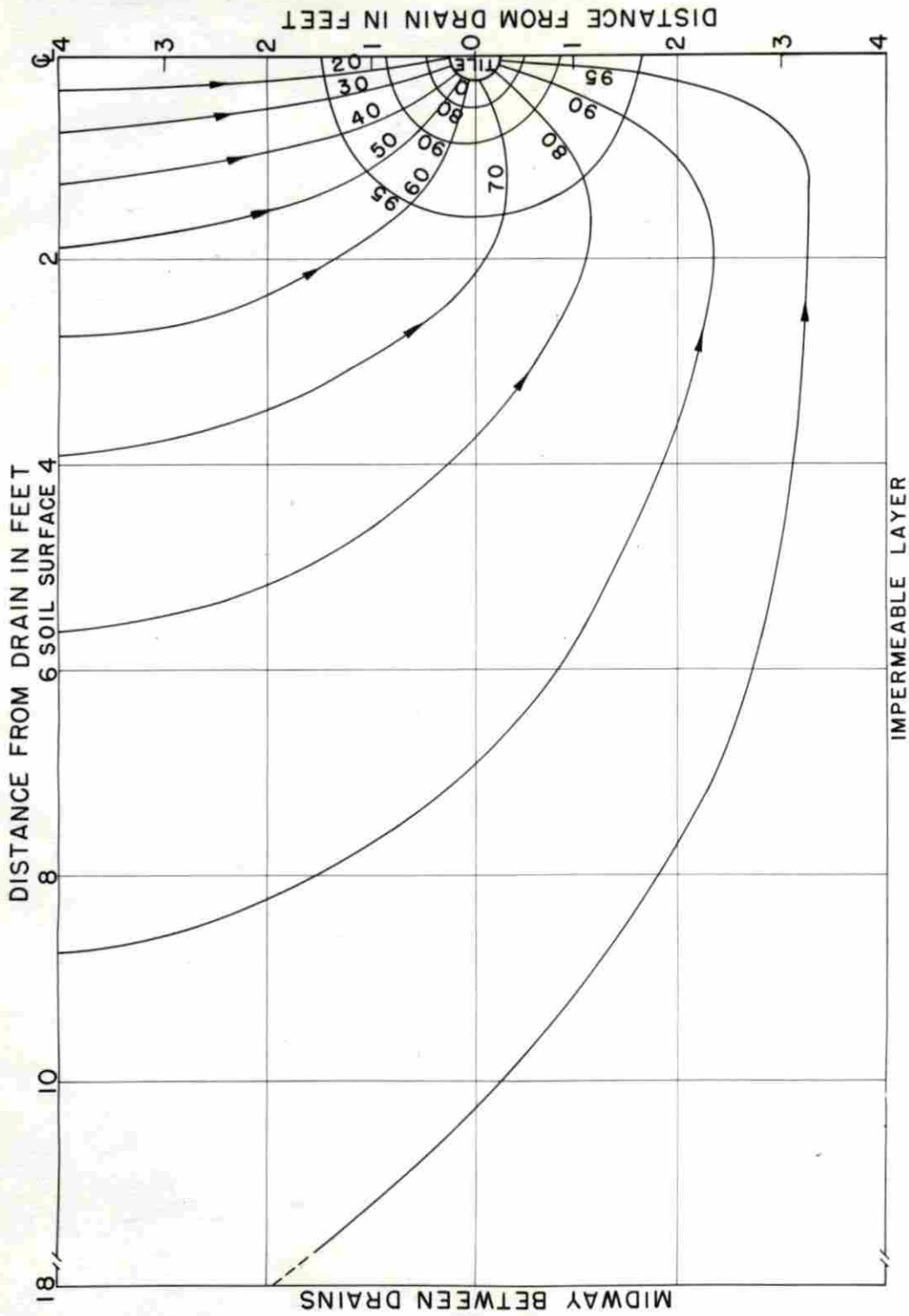


FIG. 21. Flow net of the 1/8-inch crack in the YZ plane at  $X = 0$ .

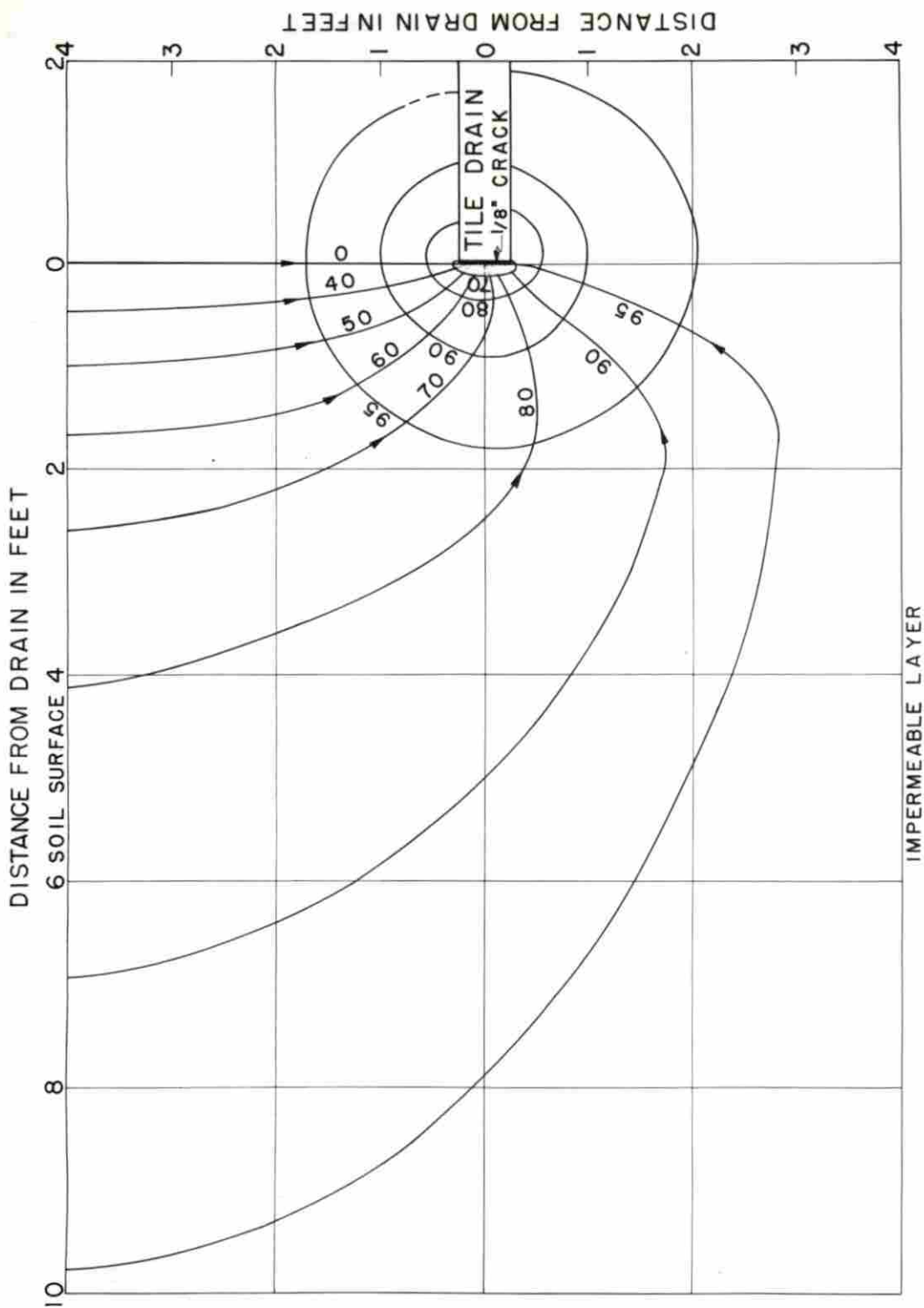


Fig. 22. Flow net of the 1/8-inch crack in the XZ plane.

crack because of the tendency for the small electrodes to corrode quickly. This caused small variations in the flow net. In the models of the 1/8-inch crack it can be seen that 80 per cent of the head loss occurs within 4 inches of the end of the drain. It was not possible to trace the 50 per cent line. Kirkham (34) and Dutz (12) have shown the tremendous drop in head or restriction offered by cracks. The loss of head caused by this restriction is dissipated rapidly and within a small radius from the drain.

The other case of practical importance which was studied was that of a gravel packed or porous end. A gravel envelope may increase the effective crack width to the extent that the tile line acts as if it were completely pervious; however, the gravel envelope does not appreciably increase the total quantity of flow into the drain. In Fig. 23 the distance to the 50 per cent equipotential line is nearly 8 inches. This is considerably more than in the case of the 1/8-inch crack. In Fig. 24 which is the gravel end in the XZ plane the 80 per cent line is 16 inches from the end of the gravel, this is nearly 3 times farther than that for the 1/8-inch crack in Fig. 22. This would seem to indicate that use of a porous end which would not allow foreign substances to penetrate it would be advantageous. The flow nets from



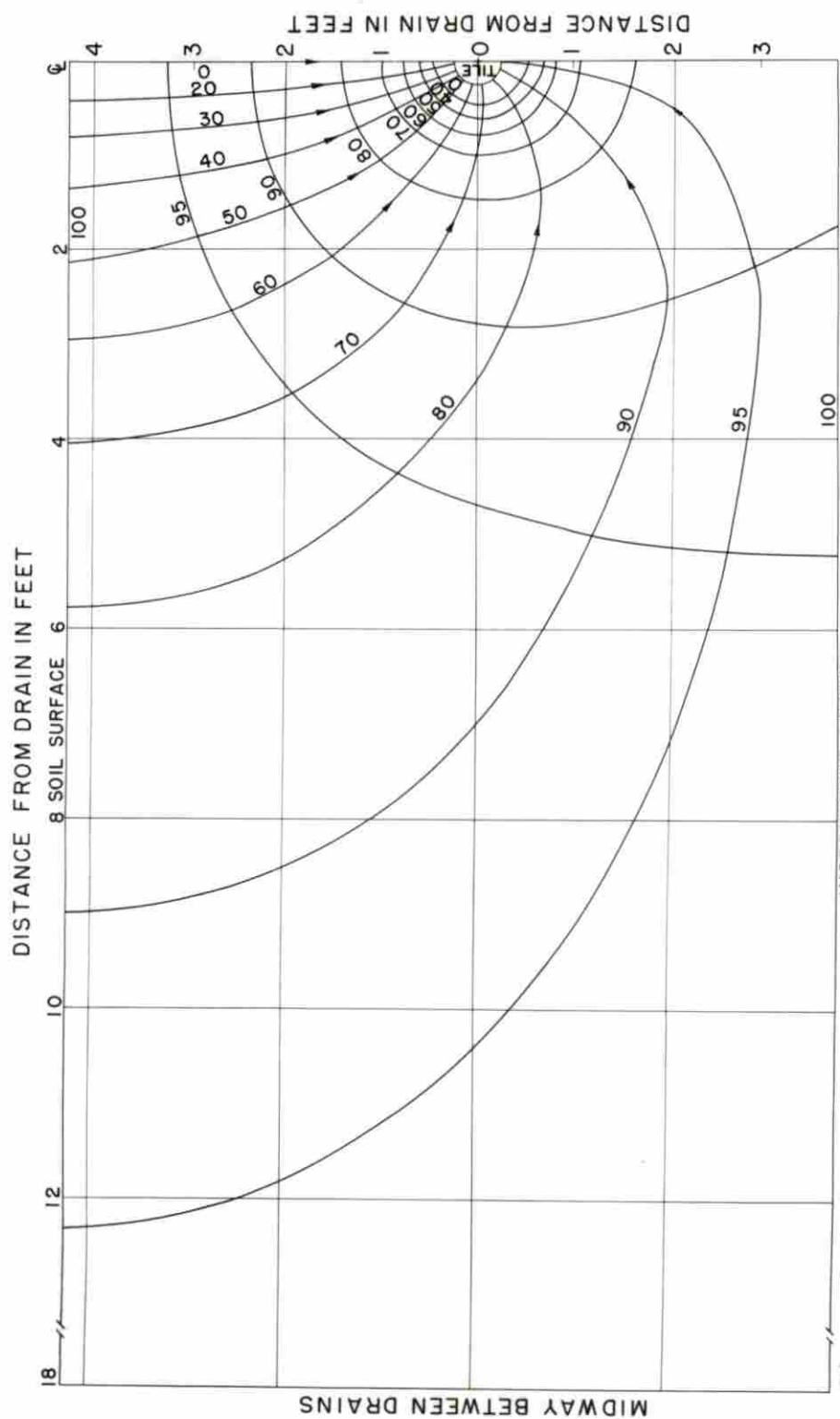


Fig. 23. Flow net of the gravel end in the  $xz$  plane at  $x = 0$ .

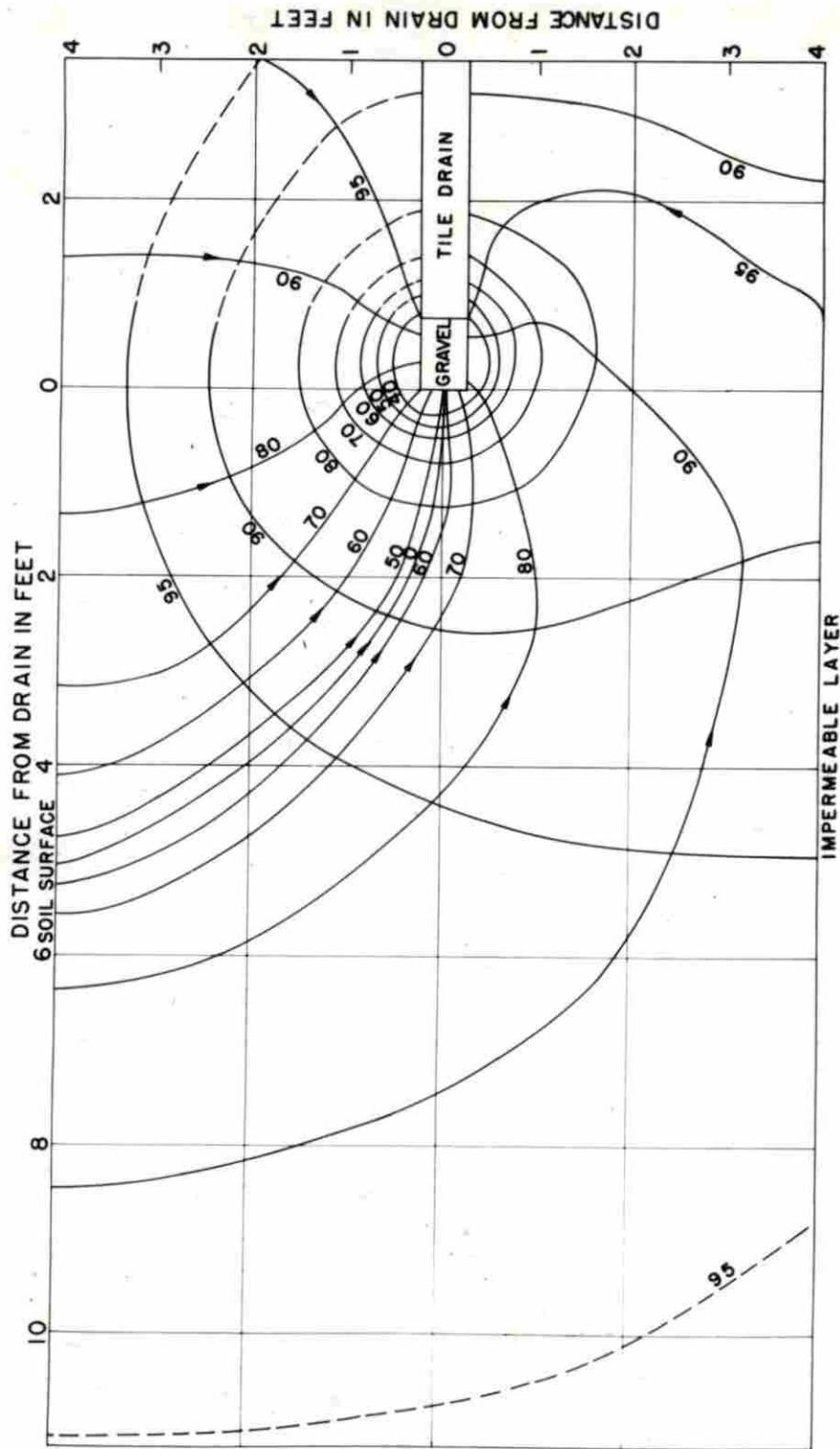


Fig. 24. Flow net of the gravel end in the  $xz$  plane.

the open end drain, which could be represented by a screen over the end of a tile, are shown in the YZ plane in Fig. 25 and in the XZ plane in Fig. 26. The open end drain is a transitional case and has little practical application.

In all previous flow nets in the XZ plane it may be noted that the streamlines, which are marked with arrow heads, intersect the surface at approximately the same distance from the end of the tile. This would indicate that the end of the tile acts as a point source except for minor variations which take place within a few diameters of the drain. The percentage of flow between adjacent streamlines for each case is the same but the quantity is greatly reduced on some models. The streamlines converge on the drain and this corresponds to an increase in velocity. It is this factor which causes most of the drop in head.

### Field Study

The determination of the true water level in field observations is quite difficult. When water table pipe are installed near a drain the water tends to converge on these pipes which act as small wells thus taking the path of least resistance. The water table becomes depressed around each

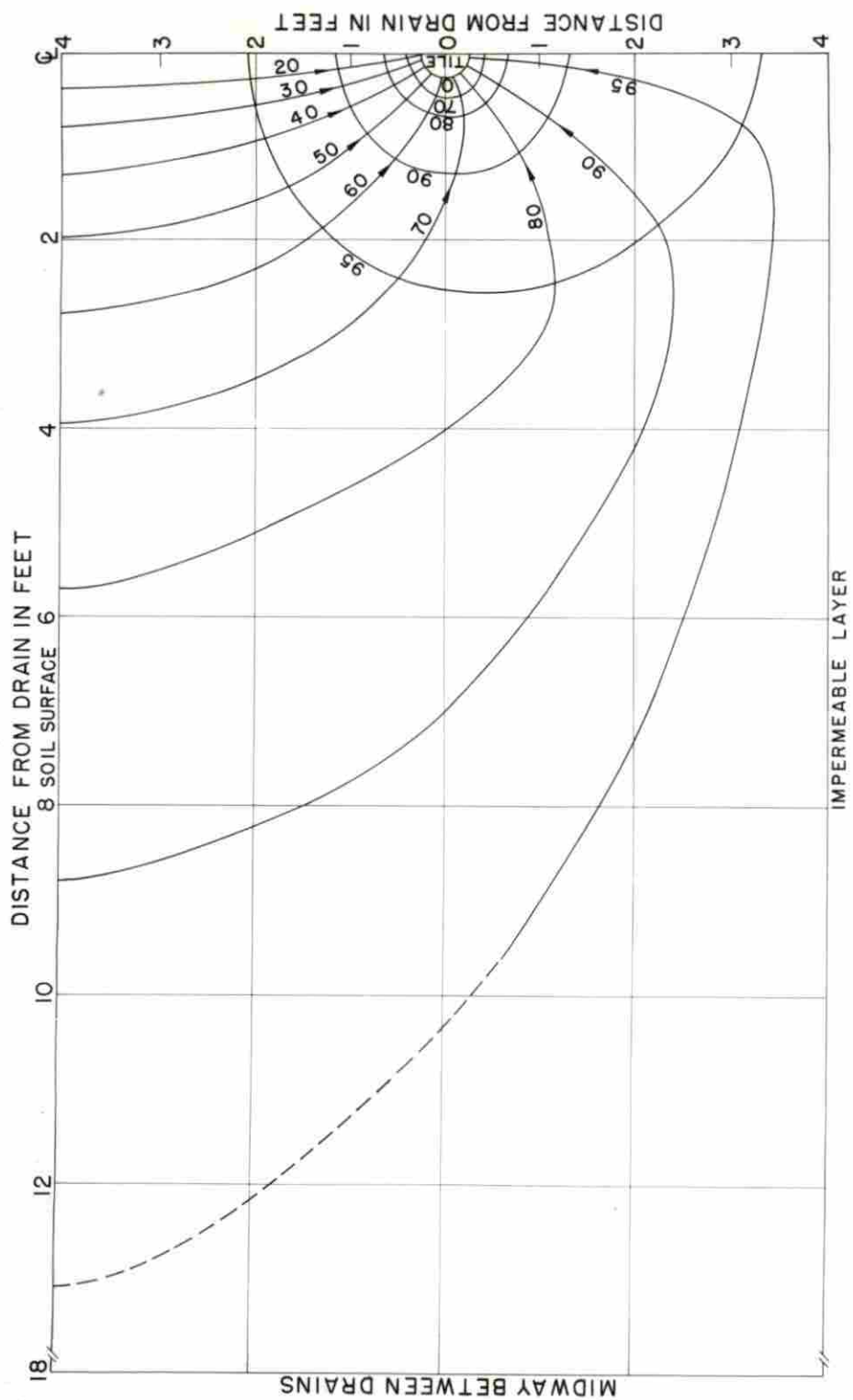


Fig. 25. Flow net of the open end drain in the YZ plane at  $X = 0$ .

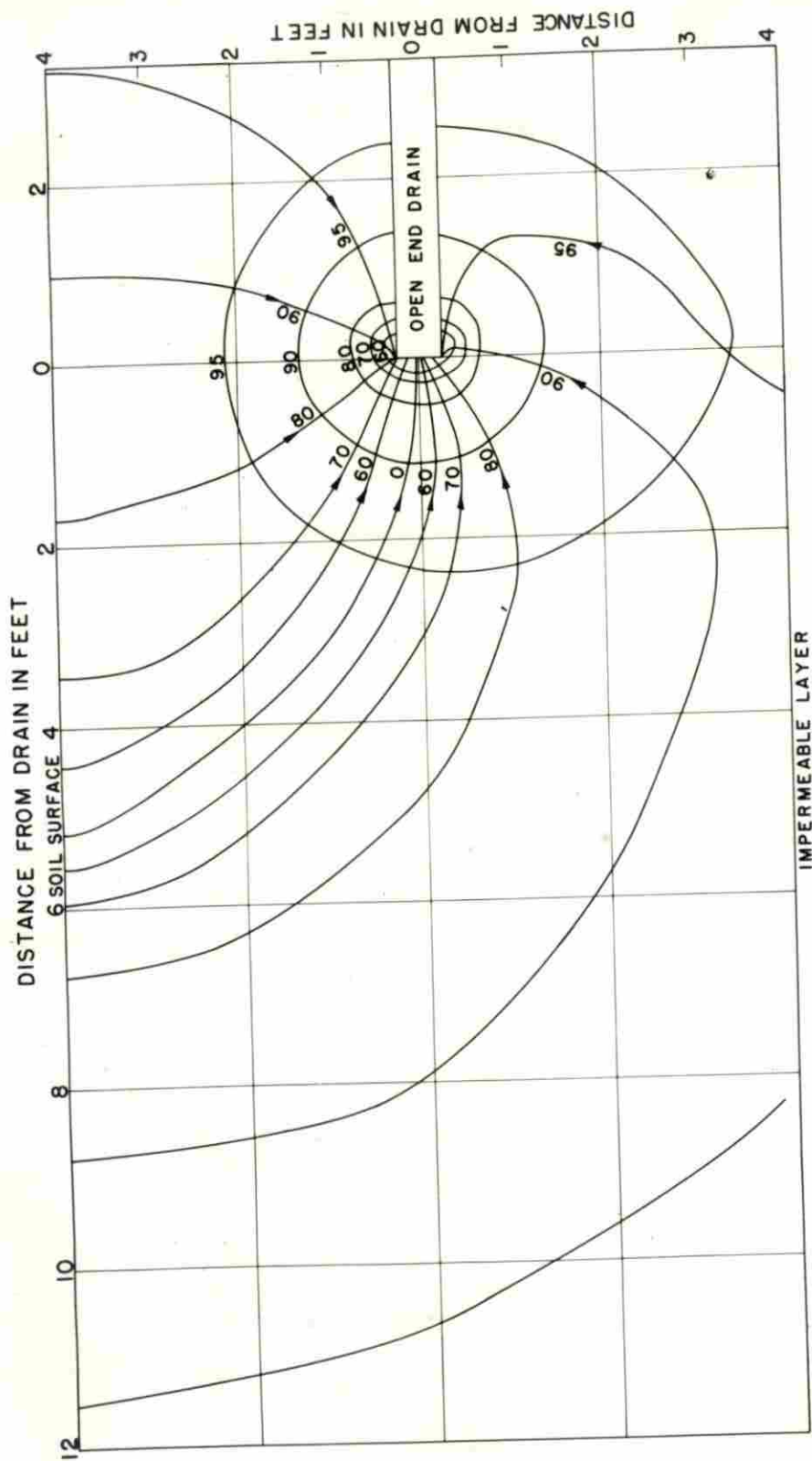


Fig. 26. Flow net of the open end drain in the xz plane.

pipe and so the true water table elevation is not obtained (32). This effect is minimized by the use of small diameter pipe. The only time when the true water table is read is when the bottom of the pipe is just even with the water table. Hore and Kidder (28) found small significant differences between large and small cased wells but concluded that this may not be important as the degree of accuracy required is not great. Water table pipe readings vary considerably in the different soil conditions. There is more difference between water table pipe readings than the lag of the water table.

Agricultural Engineering Research Farm, Ames

The data taken in the manner as described under "Procedure" was tabulated in Table 2 using the grade line of the drain as the reference plane. On examination of these data it is seen that certain of the values at 29 feet from the end of the tile line begin to fall off. This is due to the localized water table which was built up by flood irrigation. Sufficient water to thoroughly saturate the whole experiment was not available. The blank spaces on line C and line D indicate where water table pipe were not installed because of a large stone buried at this location.



Table 2. Field measurement data (Ames).

Distance from drain, feet	1	2	3	5	9	16	29
Line and run no.	Height of water table above tile grade line, feet						
A-1*	1.36	2.15	2.51	3.02	3.65	3.81	2.90**
A-2	1.19	1.81	2.06	2.32	2.90	3.15	2.78**
A-3	0.81	1.39	1.56	1.66	1.95	2.06	2.19
B-1	0.99	1.35	1.65	2.14	2.92	3.36	2.91**
B-2	0.78	1.04	1.29	1.72	--	--	--
B-3	0.88	1.04	1.23	1.52	1.65	1.76	1.80
C-1	--	1.77	2.19	2.98	3.64	3.87	3.11**
C-2	--	1.48	2.06	2.32	2.89	3.27	2.89**
C-3	--	1.21	1.54	1.68	1.96	1.97	2.07
D-1	0.87	--	2.37	2.89	3.66	3.94	3.11**
D-2	0.84	--	2.01	2.22	2.89	3.27	2.89**
D-3	0.79	--	1.59	1.69	1.99	2.14	2.14

A, B, C, D see Fig. 14 for location.

\*1 4:30 p.m., June 28, 1954.

2 6:30 p.m., June 28, 1954.

3 9:00 a.m., June 29, 1954.

\*\*Values falling off due to localized water table.

The data obtained from three runs were plotted. Fig. 27 shows the water table shape at 4:30 p.m. June 28, 1954. The location of the lines are shown in Fig. 14. It can be seen that line B is much lower than the other lines. This indicates that the water table is much lower at this position. There is very little difference between the other lines, however, these water table surface lines follow very closely the equipotential lines for comparable positions in Fig. 18. At a short distance from the drain the D curve is slightly higher indicating that it does not drain as far as the A curve. Curve C is an intermediate curve at 45 degrees to D.

Figs. 28 and 29 are plots of the data from Table 2 for 6:30 p.m. June 28 and 9:00 a.m. June 29, 1954. In both of these figures it is seen that line B is much lower than the other three lines. The difference is not as great because of the lower water table and reduction in head.

If a point within 20 feet of the drain is selected on curve B of Fig. 27 and an average value secured from the other three curves A, C, and D for the same elevation, it will be seen that in using the distance from the end of the drain as a reference the point selected on curve B will be about twice as great as that selected from the

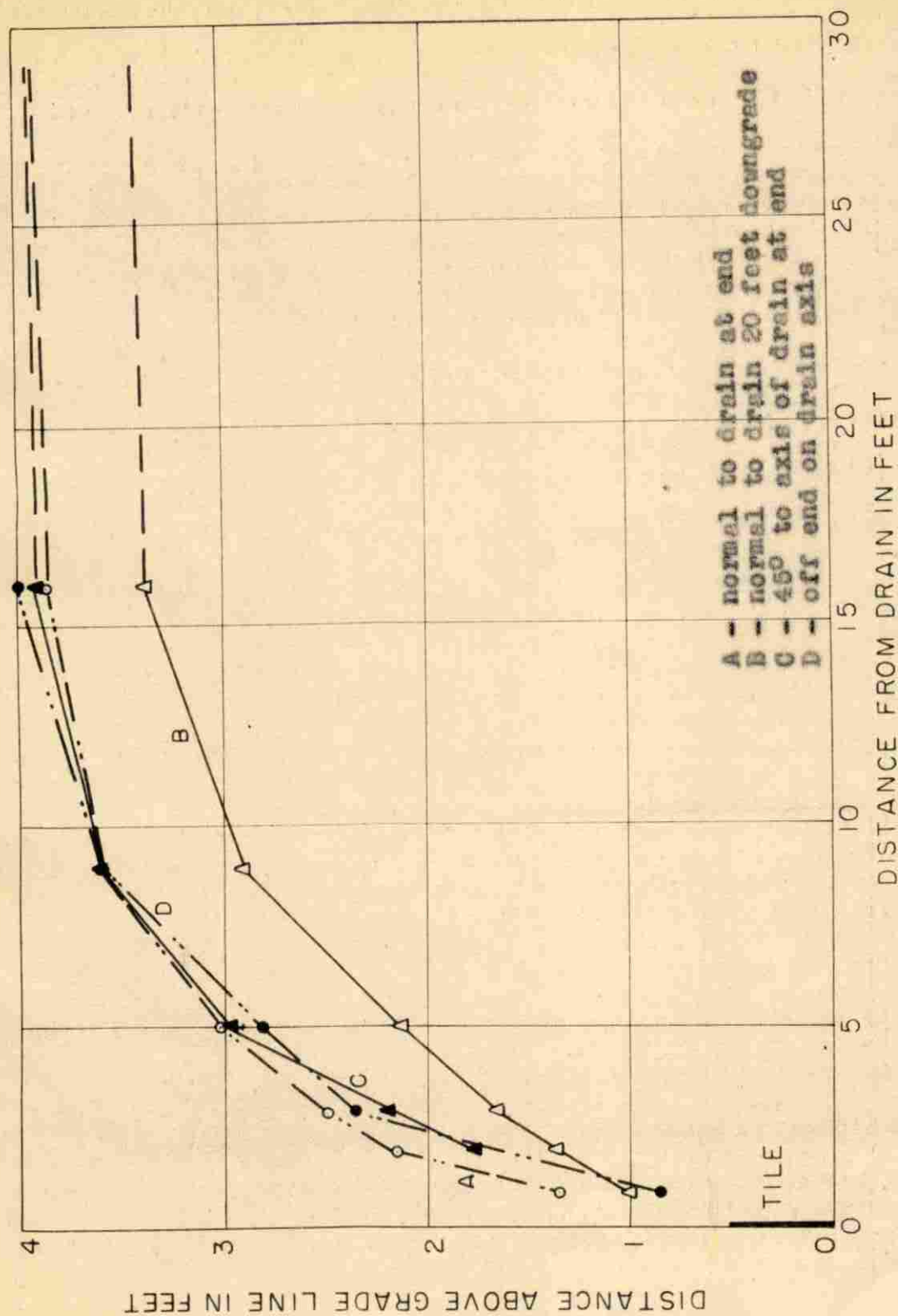


Fig. 27. Water tables near the end of a tile line at Ames, 4:30 p.m., June 28, 1954.

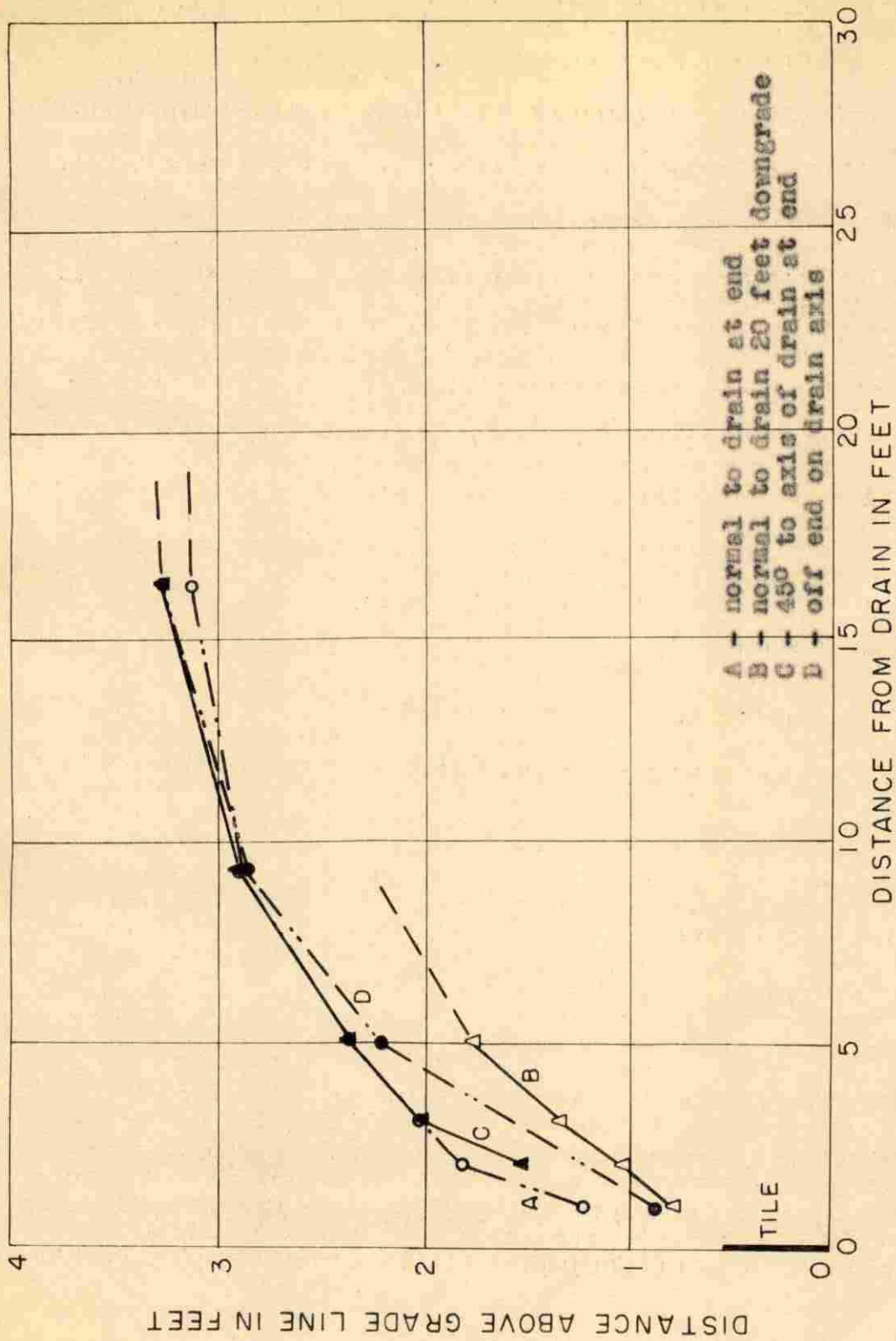


Fig. 28. Water tables near the end of a tile line at Ames, 6:30 p.m., June 23, 1954.

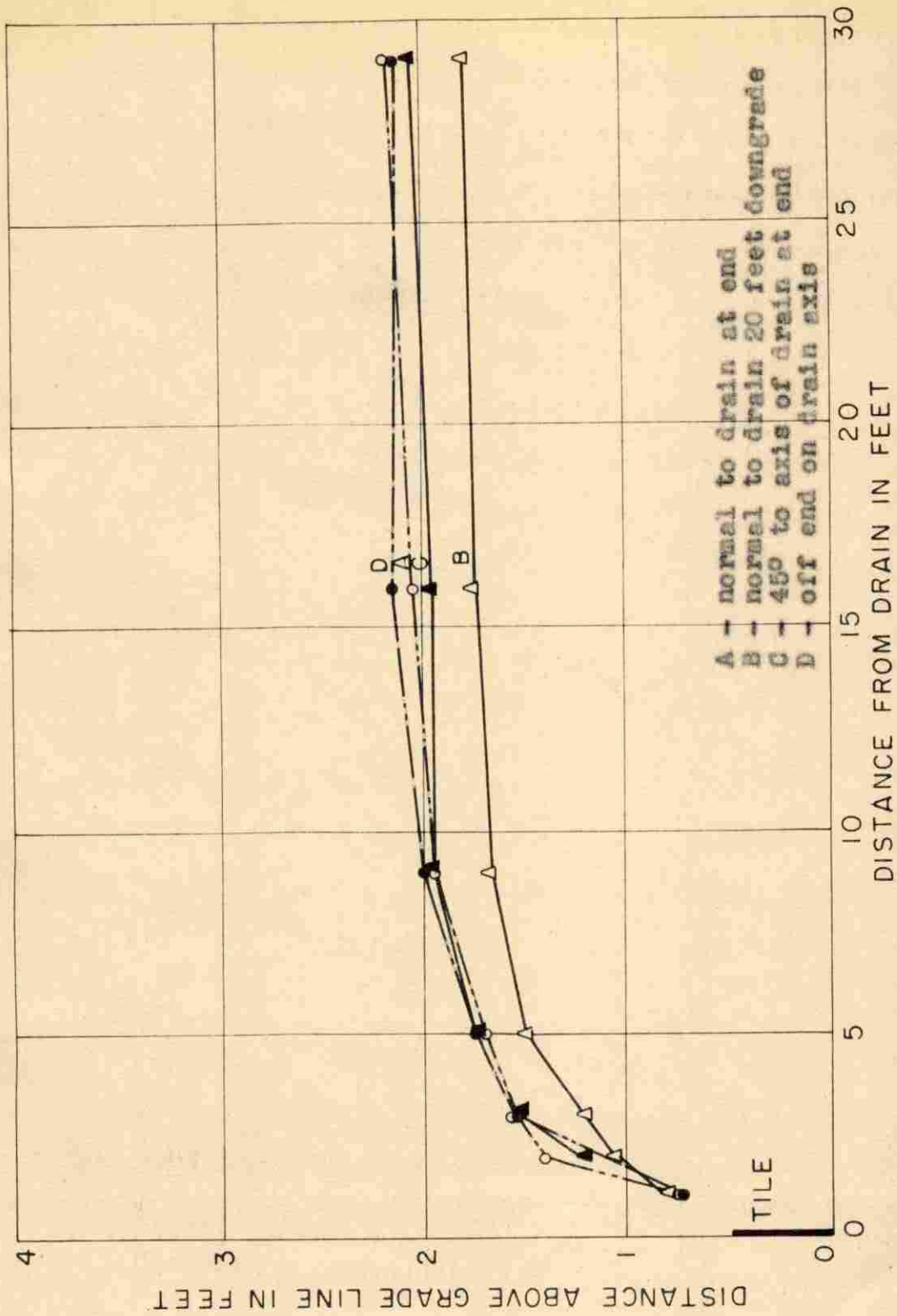


FIG. 29. Water tables near the end of a tile line at Ames, 9:00 a.m., June 29, 1954.



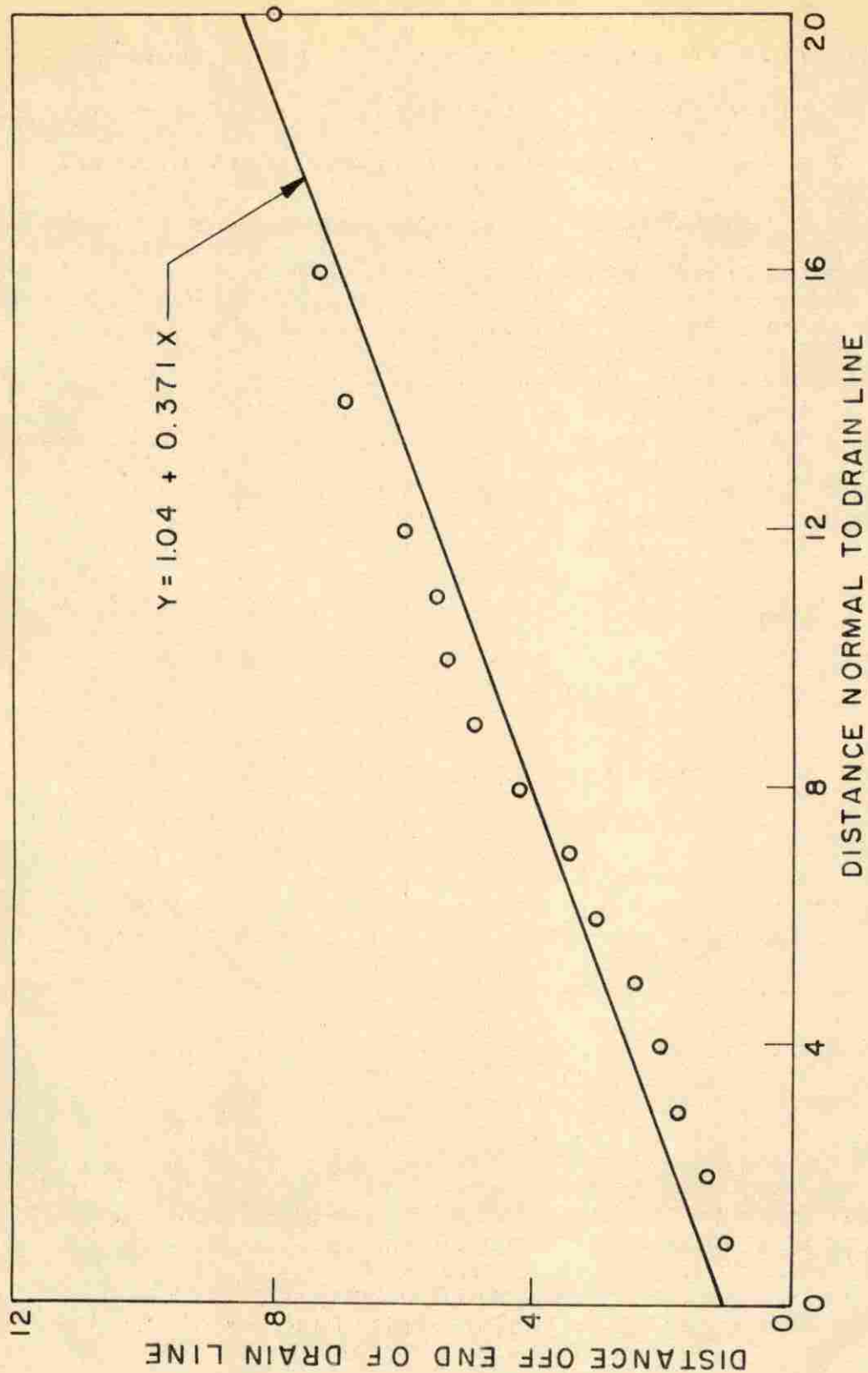
average of the three curves. This method was followed and the open circles in Fig. 30 show the points for Fig. 27. The regression line  $Y = 1.04 + 0.371X$  was plotted which gave a  $t$  value of 6.63. The  $t$  value for 13 degrees of freedom at the 1 per cent level is 3.012. This is highly significant. For example if an underdrainage system gives adequate drainage for a spacing of 40 feet at a point 20 feet downgrade from the end of the lateral then the water table level 20 feet off the side of the drain will be the same elevation as the water table 10 feet off the end of the drain. An interpretation would be for close spacings, equal elevations are only half the distance off the end as off the side of the drain.

Clarion-Webster Soil Association Experimental Farm, Kanawha

The experiment conducted at Kanawha was to measure any water table differences between the end of a tile line closed off with concrete and the end of a tile line which had a porous copper plate and a sand backfill thus giving a porous end. The results of the model study as illustrated in Figs. 22 and 24 indicated that there should be a measurable difference. The sand backfill being superior to the plugged end with regard to the time required to lower the water table off the end of the lateral drain.

On June 18 and June 21 heavy rains totalling 10 inches were received on this experiment. The data obtained from the water table observations are summarized in Tables 3 and 4. The height of the water table is based on the grade





**Fig. 30.** Comparison of distance off end of drain with distance off side of drain, for equal water table levels.

Table 3. Water table levels (Kanawha).

Run no.	1	2	3	4	5
Date	6-18	6-21	6-22	6-23	6-24
Pipe no. Height of water table from tile grade line, feet.*					
Copper plate with sand backfill					
62	1.80	1.90	1.60	1.40	1.20
65	2.33	2.40	1.94	1.60	1.30
610	3.10	3.28	2.47	1.90	1.50
42	2.65	2.68	1.93	1.40	1.30
45	3.00	2.95	2.10	1.55	1.30
410	4.20	3.93	2.47	1.81	1.55
22	3.50	3.40	1.80	1.46	1.30
25	3.90	3.60	2.15	1.45	1.33
210	4.10	3.80	2.00	1.10	1.50
End plugged with concrete					
52	1.50	1.64	1.17	0.95	0.69
55	2.10	2.08	1.26	0.95	0.77
510	3.20	2.83	1.60	1.25	0.98
32	1.80	1.90	1.10	0.80	0.50
35	2.53	2.47	1.63	1.00	0.80
310	3.20	3.20	1.70	1.40	1.20
112	2.00	1.80	1.50	1.35	1.30
15	2.33	2.49	2.05	1.70	1.57
110	3.63	3.03	1.92	1.45	1.27

\*Average of 2 pipe

Table 4. Average water table levels (Kanawha)

Run no.	1	2	3	4	5
Date	6-18	6-21	6-22	6-23	6-24
Distance of pipe from end of drain, feet	Height of water table from tile grade line, feet				
Calculated average for copper plate with sand backfill*					
2	2.64	2.66	1.77	1.42	1.30
5	3.07	2.98	2.03	1.53	1.30
10	3.80	3.67	2.31	1.60	1.35
Calculated average for end plugged with concrete**					
2	1.77	1.78	1.26	1.03	0.80
5	2.33	2.25	1.66	1.22	1.04
10	3.34	3.02	1.79	1.37	1.15

\*2 foot pipe is average of 52, 32, and 112 from Table 3

5 foot pipe is average of 55, 35, and 15 from Table 3

10 foot pipe is average of 510, 310, and 110 from Table 3

\*\*2 foot pipe is average of 62, 42, and 22 from Table 3

5 foot pipe is average of 65, 45, and 25 from Table 3

10 foot pipe is average of 610, 410, and 210 from Table 3

Rainfall: June 18 - 4.25 inches  
 June 19 - 1.67 inches  
 June 20 - 0.62 inches  
 June 21 - 3.51 inches

line of the tile drain and is measured above this point. Values in Table 3 are averages of 2 pipe at each location and values in Table 4 are averages of 6 pipe in comparable locations on 3 replicates. The averaged values from Table 4 are plotted in Fig. 31.

In Fig. 31 it should be noted that the water tables are relatively flat. The 2-foot and 5-foot pipe are located in the old trench and even here there is no marked difference in the flat shape of the water table. It must also be noted that in every case the elevation of the water table is much higher where the tile line had the copper plate and sand backfill treatment. The rate of drop of the water table at 10 feet from the drain is much faster with the sand backfill. Part of this may be due to the increased head as it is higher than the head over the ends plugged with concrete.

The results obtained were not expected as they did not follow the model study results. The model study indicated that the laterals with the sand backfill would drain much more quickly than would the laterals with the concrete ends. The plotted data in Fig. 31 shows the reverse has taken place with all runs being consistent. The water tables over the sand backfill continues to be higher from the first day to the last.

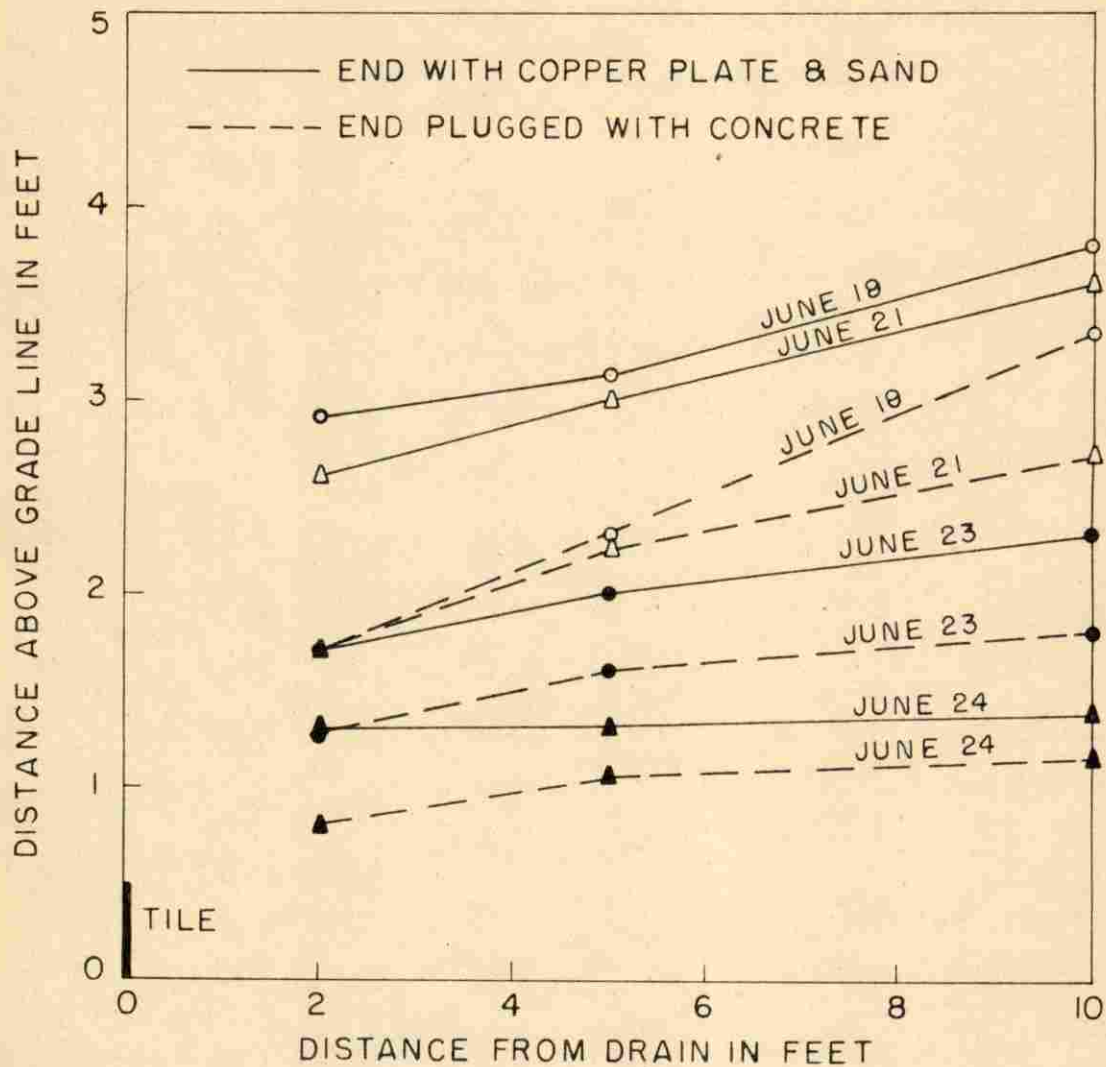


Fig. 31. Water table positions off the end for two types of end closures (Kanawha).



An interesting conjecture regarding the results is that following the heavy rain of 3.5 inches on June 21 some of the land was ponded while the remainder was not; this would account for some of the values, such as the water table levels in series 42 and 22, being high. These inconsistent values would not materially affect the final result so the reason for the above results must be elsewhere. It is possible but not probable that there are soil differences which would cause it. The outlet of the tile system was submerged for a period of four days. This would cause a small back pressure in the drain even though the difference in elevation is considerable. There is also the possibility that the 5-inch main tile A-5.1 was running full which would cause a back pressure in the laterals. If the water table elevation was sufficient to overcome the back pressure by a small amount, then drainage would take place. The laterals with the sand backfill would transfer this back pressure to the surrounding soil more easily and thus retard the drop of the water table. The fact that lines A-5.16 and A-5.15 were practically the same on the first day but diverged further apart the next day would also indicate that the lines were running under back pressure. These two lines have the highest elevation and so would feel the effect of back pressure last. If any portion



of the system is to run full which tends to cause a back pressure, then a porous backfill would not show up the theoretical advantage for this system.

The data in Table 5 was gathered for another experiment and was not specifically designed for use in this study; however a few points are noted which have application. In Fig. 15 it is seen that pipe 56 is located midway between A-5.16 and A-5.15 at the end of the line. Pipe 1136 is located 100 feet downgrade from this pipe and 1123

Table 5. Water table levels (Kanawha).

Run no.	1	2	3	4	5
Date	6-18	6-21	6-22	6-23	6-24
Pipe no.*	Height of water below ground surface, feet				
56	0.4	0.35	1.12	1.81	2.36
1136	2.72	1.42	2.13	2.69	2.99
1123-5**	--	--	2.03	3.40	3.56
1123-50	--	--	3.36	3.10	3.20
23	0.00	0.05	2.12	2.91	3.24
2146-a	--	1.80	2.82	3.19	3.29
12	0.03	0.00	0.74	1.89	2.48
2146-b	--	1.10	2.13	3.22	3.40

\*Each pipe is an average of two readings

\*\*1123-5 is 5 feet from the drain, remainder of pipe 50 feet from drain

located 200 feet downgrade from 56. Pipes 23 and 12 are located even with the end of the tile line and between A-5.13 and A-5.12 and between A-5.12 and A-5.11 respectively. Pipes 2146 are located 200 feet downgrade from pipes 23 and 12.

It may be seen from Table 5 that the water level in the pipes even with the end of the tile line are higher than the water level in pipes which are downgrade from the end of the tile. The difference ranges from 2.5 feet on June 18 to 0.63 feet on June 24. The average difference is 1.05 feet. This would indicate that even with the end of a drain line the point at the midspacing is not as well drained as further downgrade.

On June 22 it should be noted that the 1123 pipe located 5 feet from the drain has a higher water level than does the pipe at the center of the spacing. This may be due to the same phenomena which has caused the difference in water table elevations at the end of the drain.

#### Howard County Experimental Farm

The results of the experiment at Howard County tended to corroborate the results obtained at Ames. The soil is

quite variable and difficult to drain in many places. The water table did not reach the surface at this location, but data were obtained from following a relatively high water table after the heavy rain of June 22, 1954. Fig. 32 is plotted from the data in Table 6. It can be seen from this figure that the water table is very flat except for 16 feet close to the drain. From the drain to 16 feet along line A the water table rises rapidly and then flattens out to the midpoint of the spacing. An explanation of this may be found in the fact that there was probably deep seepage on this location. Kirkham (32) working with piezometers on the same location found that the hydraulic head decreases with depth which indicated deep seepage. Van Deemter (76) indicated that deep seepage may have more effect upon the water table than the drain itself.

In Fig. 32 it can be seen that the water table pipe off the end of the drain follow the same pattern as the water table pipe normal to the drain. The minor differences here are due to there being no pipe at 10 feet normal to the drain at the end. This follows very closely the results obtained at Ames as shown in Fig. 27.

At the midpoint of the spacing the water table in the pipes in line E (Fig. 16) which were 6 feet apart was the same except for minor differences. This can be seen by

Table 6. Field measurement data (Howard County).

Run no.	1	2	3	4	5
Date	6-22	6-23	6-24	6-25	6-26
Pipe no. Height of water table above tile grade line, feet					
E-7	3.10	2.77	2.84	2.83	2.45
E-6	3.16	2.84	2.73	2.61	2.51
E-5	3.09	2.75	2.57	2.44	2.38
E-4	3.06	2.71	2.55	2.41	2.38
E-3	3.12	2.77	2.75	2.54	2.47
E-2	3.16	2.83	2.65	2.51	2.42
E-1	3.09	2.82	2.65	2.52	2.45
E	3.65	3.13	2.95	2.80	2.71
E-11	4.00	3.20	2.84	2.59	2.42
19c	2.48	2.36	2.18	2.12	2.08
D-2	2.03	1.73	1.52	1.39	1.33
D-5	2.32	2.02	1.85	1.74	1.67
D-10	2.82	2.57	2.39	2.24	2.16
D-16	2.93	2.61	2.41	2.21	2.07
A-5	2.30	1.95	1.75	1.55	1.52
A-16	2.95	2.68	2.55	2.48	2.45
A-42	3.05	2.71	2.55	2.41	2.38

Rainfall: June 16 - 1.90 inches  
 June 18 - 0.82 inches  
 June 19 - 0.50 inches  
 June 20 - 0.97 inches  
 June 21 - 0.85 inches

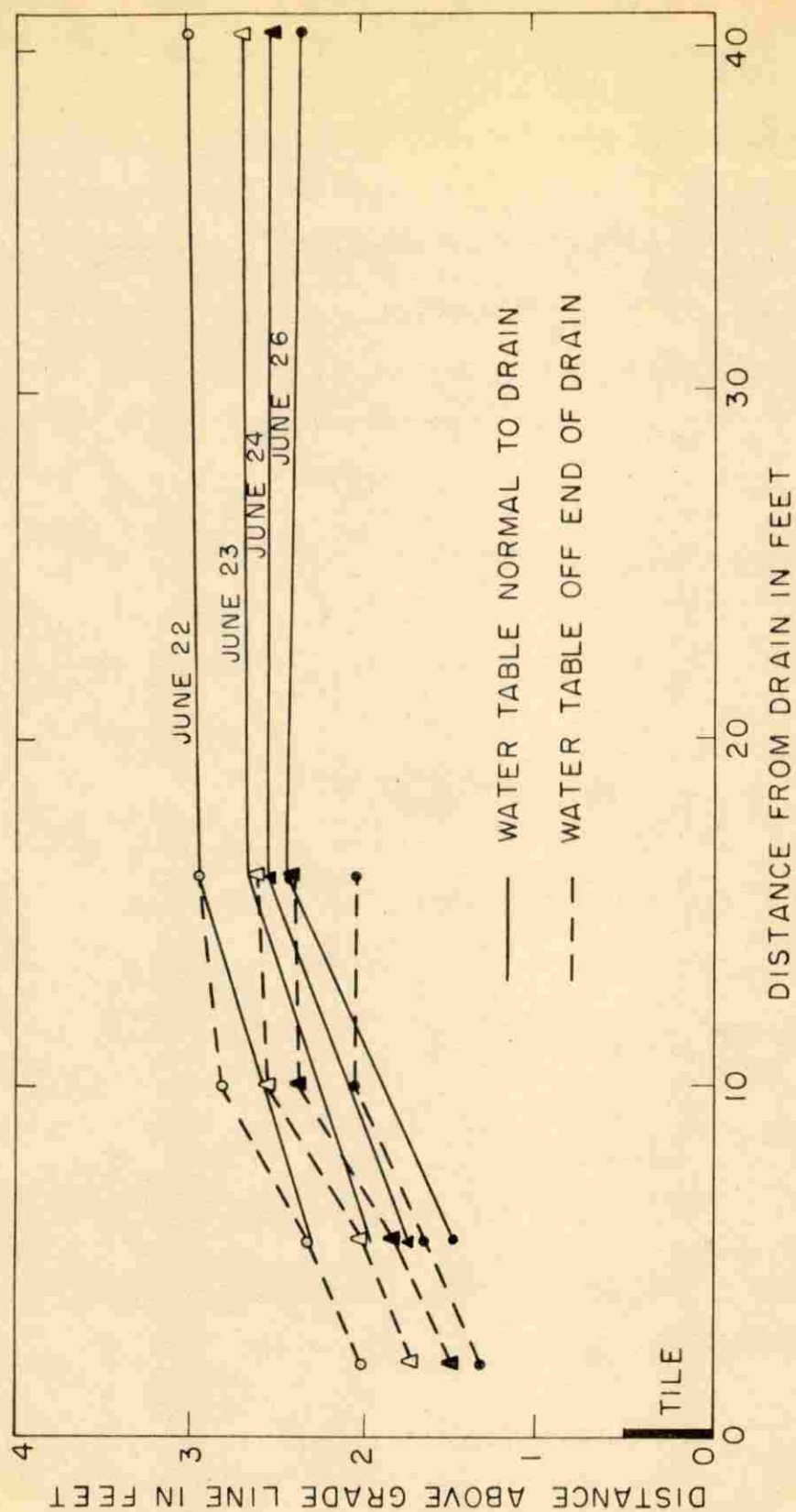


Fig. 32. Water tables near the end of a tile line at Howard County.



inspection of Table 6. The reason for the difference in pipe E is that it is located 50 feet downgrade from the end of the tile and is at a lower elevation when based on the grade line of the drain. At pipes E-11 and 19c located 130 feet downgrade from the end of the tile there is a difference of 1-1/2 feet in the water table as compared with 3/4 foot at the upper end (pipes A5, E4). This difference can be explained by upward seepage from the side of the hill at the lower pipes and deep seepage at the top of the hill.

#### Miscellaneous observations

Because of limited rainfall and low water tables an attempt was made to analyse some water table data obtained in 1947 at the Crystal Lake Experimental Farm. These data showed that the water table was quite flat on this peat soil which indicated that their pump was inadequate to handle the influx of water from higher land. The position of the critical pipe was located on a hillside with a variable soil type. This data was of little value for this problem.

An attempt was made to establish a water table by means of sprinkler irrigation. The Ankeny Field Station



was selected for the site as there was an ample supply of water available from a fire hydrant. The water was conveyed 750 feet in fire hose with a considerable drop in pressure. Two lateral lines 40 feet apart and 100 feet long were used with 12-inch risers and 3/16- x 1/8-inch nozzles on a No. 40 Rainbird sprinkler head. It was possible to build up a local water table but due to the crop of young soybeans and the slope of the land the water table was not of uniform height. Water ran down the slope and built the water table higher in the center, and the edges were then very low where the mound of water dropped off. It was practically impossible to analyse the data from this experiment as there was no way of establishing whether the effect was due to the tile system or the fact the water table was localized and not predictable.

## DISCUSSION

The results obtained from the model study were consistent and could be reproduced, however, accuracy could be improved by the use of better equipment. The electric analogue functioned very well near the top of the tank but close to the model and near the bottom of the tank it was increasingly more difficult to obtain good accuracy. The difficulty seems to be in the recording and probing device. A more rigid probe which could be moved in any direction is to be desired (see reference 71). For more accurate work near the model the probe should be reduced in size. A difference was found in setting up the correct bridge ratio. The ratio as used by Dutz (12) did not give the same results as the ratio used here. It is possible that the difference in resistances used for the same per cent line changed the current which affected the exact location of the equipotential lines. There was an actual difference in the model tank of approximately 1 inch on the 80 per cent line at the water surface.

The flow nets obtained from the model study show that there should be measurable differences between a crack and a gravel end. The field data did not prove this and so it is possible a complete answer is not available at this time

for the proper interpretation of the flow nets which were derived by using a wire for a boundary (Fig. 20) off the end of the tile drain. The shape of these flow lines in Fig. 20 is probably correct, however the numbering of these lines may require further study. Muskat (49, p. 184) shows a line source does not have this bunching up of streamlines at the end. Further work that might be pursued by a model study would be using two drains in place of one to study the pattern between the tile lines. It is also advisable to use several cracks to get a cumulative effect which was not achieved here. The other phase of the model study which should be investigated is that of finding a mathematical solution to the problem of flow into the end of a tile drain.

The field study was severely handicapped by low water tables. The natural water table at Ames, Ankeny, and Howard County was not high enough so that more data should be gathered in the future to corroborate the results obtained at this time. The actual sites of the experiments might also be relocated to get more uniform soil conditions. The results of the experiment at Ames were quite satisfactory but was limited to too small a spacing because of the inability to build an extensive water table. The same experiment should be carried on where it is possible to continue

the water table pipe for a greater distance both normal to the drain and out from the axis of the drain. This should result in an improved Fig. 30, by which it is possible to forecast the distance off the end drained in terms of proper spacing. Due to the deep seepage at Howard County it was not possible to find any differences in water table elevation along the midpoint of the spacing. There was an indication there might be a difference at Kanawha. This could be pursued further by setting pipe along the midspacing line. The solution to this problem is important in laying out experimental plots.

The case of the sand backfill needs much more study. This same treatment should be placed on other lines to determine the cause of the failure to follow the predicted course. Piezometers should be placed in the tile line to determine back pressure, and at midspacing to determine deep seepage.

It is very difficult to build an artificial water table by the application of water to the surface of the soil. The time required to fully saturate the soil is considerable. The rate of application must be very low or there is too much runoff. The infiltration must be high which can only be achieved by a good sod on practically flat grades. The application must be carried on at least 50 feet past the experiment. There must be a good source of water to thoroughly saturate the soil.

## SUMMARY AND CONCLUSIONS

The purpose of this investigation was to determine the water table shape at the upper end of subsurface drains by means of model studies and field observations. Past literature contains very little information regarding the distance the upper end of a tile drain will affect the water table. This information is of value in the design of drainage systems to economize on tile. A more important use of this information would be in the design of experimental plots where space is important. Past practice has been to leave bulk areas near the end of the tile line so that any detrimental effect of higher water tables would not affect the experiment. A limited study was made on two methods of closing off the end of a tile drain. The methods studied were an end closed off with concrete and an end closed off with a porous plate and surrounded with sand.

The effect of various methods of closure was studied in the laboratory by the use of an electric analogue. Flow nets were obtained from both direct and inverted analogues. One-inch copper tubes were used for the drain tiles which represented a 6:1 model scale. The results showed that for a porous filter at the end of a tile drain there should be

a small area in the field 7 feet from the end of the tile where most of the volume of flow enters the soil surface. This has not been observed in the field. The reason for this is possibly the use of an artificial boundary in tracing the streamlines which introduces undetermined errors. A comparison was made between a 1/8-inch crack at the end of the drain and a porous end. It was observed that the same loss of head took place three times further from the drain for the porous end than for the case of the 1/8-inch crack. The electric analogue gives a maximum effect which can be expected as it is used for the case of ponded water and this difference in head loss advantage for the porous end may be considerably reduced under actual drawdown conditions. There is an indication from the model study that there is a theoretical advantage in using a porous end. The model studies also revealed that the loss of head is greater off the end of the drains than off the side normal to the axis of the drains.

The results of the model studies were tested at various locations in the field. At Kanawha the field tests after very heavy rains gave higher water levels over the drains that had a porous filter than over the ends of the lines which had been closed off with concrete. This is the reverse of the model results and might be explained by the probability of high back pressure in the tile lines or soil



variability. The rate of drop of the water table was greater over the porous end. At the same location a series of water levels parallel to the drains and at the midpoint of the spacing showed that the water table was about 1.05 feet higher at the upper end of the tile line than at a distance downgrade. The same experiment at Howard County showed no difference in water levels. This may be explained on the basis of previous experiments which indicated deep seepage at this location.

An experiment at Ames to determine the shape of the water table around the end of the tile line showed that the field conditions followed the results of the model studies very closely. The same observations at Howard County also agreed with these results close to the drain. At a selected distance from the tile the water table at a distance of 20 feet downgrade from the end of the tile line was much lower than at the end of the tile. This is based on observations of three drain lines and agrees with the model studies. It was found that close to the upper end, equal water table levels are semicircular with a radius about one-half the distance the same water level is from the drain at a point 20 feet downgrade.

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